Adaptive Kernel Estimators for Population Abundance Using Line Transect Sampling

By

Baker Ishaq Albadareen

Supervisor

Dr. Omar Eidous

Program: Statistic

June 26, 2011

Adaptive Kernel Estimators for Population Abundance Using Line Transect Sampling

By

Baker Ishaq Albadareen

B.Sc. in Actuarial Sciences, University of Jordan, 2006

Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science in the Department of Statistics at Yarmouk University, Irbid, Jordan.

Approved by:

Dr. Omar M. Eidous .... (Chairman)

Associate Professor of Statistics, Yarmouk University

Dr. Abdullah A. Smadi .... (Member)

Associate Professor of Statistics, Yarmouk University

Dr. Raed Al-Zgool .... (Member)

Assistant Professor of Statistics, Al-Balqa University

June 26, 2011

DECLARATION

I hereby declare that the work in this thesis is my own work has been in accordance with generally accepted scientific bases.

June 26, 2011 BAKER ISHAQ MOHAMMAD ALBADAREEN

ACKNOWLEDGMENTS

In the name of Allah, Most Gracious, Most Merciful, I want to say "The Last prayer is praise be to Allah, Lord of the words" who teach Human beings that the Human being does not know, and whatever we do remains perfect for Allah.

I would like to say thanks to my supervisor Dr. Omar M. Eidous for his non-stopping support, encouragement and efforts to fulfill this thesis and I appreciate his time to teach me the process and the way of writing a research, also I appreciate his long patience to me on all steps of the research. Also, I would to thank the committee members: Dr. Abdullah A.Smadi and Dr. Raed Al-Zgool to give their time in reading and reviewing this thesis including the constructive criticism that helped me to rectify some ideas.

I would to thank also all doctors and professors in my second home "the department of statistics at the Yarmouk University" for their Excellent efforts to teach me the science of statistics, especially the courses that are related with data processing such as linear regression, simulation using various statistical packages which gives an indicator how they were care about the technological revolution in the statistics field and the probability theory and the mathematical statistic. Also, best thanks for my studied and work friend. Especially, Mr. *Madallah Almahameed*, and Mr. *Nael Hendi* for their limitless support.

I never forget my best friends Mr. *Anas Al-akhras*, and Mr. *Abdel Rahim Al-atrash* who advised me to continue in my study after we had achieved together the bachelor degree in Actuarial science from the University of Jordan.

Most of all, love and appreciations goes to my father and my mother who never stops supporting me and doing their best to make me achieve this level of education. Also, I want to thank my wife and all my brothers and my sisters for their continuous prayers and encouragements.

TABLE OF CONTENTS

LIST OF TABLES

Table No. **Page**

Table No.		Page
3.3	The point estimators of $f(0)$ for the stakes data set when the sample size is 68 and the true values of $f(0)$ is 0.110294.	54
3.4	The point estimators of $f(0)$ for the stakes data set when the sample size is 67 and the true values of $f(0)$ is 0.110294	55
3.5	The point estimators of $f(0)$, D, relative bias (RB), relative mean error (RME) and the standard error of each estimator of $f(0)$ for the Hemingway data set. The bandwidth $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}} = 58.209$, and $n = 73$. The true values of $f(0)$, D are unknown	56

LIST OF FIGURES

Figure No. **Page**

LIST OF ABBREVIATIONS

K	The kernel function.
h	The bandwidth value used in the kernel estimators.
AMSE	The asymptotic mean square error.
EP	The exponential power model.
HR	The hazard rate model
BE	The beta model
EFF	The efficiency of some estimator.

ABSTRACT

Albadareen, Baker Ishaq. Adaptive Kernel Estimators for Population Abundance Using Line Transect Sampling. Master of Science Thesis, Department of Statistics, Yarmouk University, 2011 (Supervisor: Dr. Omar Eidous).

This thesis introduces variety of adaptations of the classical kernel estimator using line transect sampling. Several new estimators are proposed and the simulation technique is adopted to compare the performances of these estimators with respect to the classical kernel estimator aiming to identify the most promising estimator. Moreover, other adaptations of the resulting estimators are devoted to correct the negative bias that associates the classical kernel estimators. Some applications of these estimators on real data sets are addressed and discussed.

Keywords: Kernel Method, Line Transect Method, Nonparametric Estimators, Bandwidth, Simulation.

الملخص

البدارين، بكر اسحق، " تحسين تقادير النواة لكثافة المجتمع باستخدام معاينة الخطوط العرضية" رسالة ماجستير بجامعة اليرموك، 2011. (المشرف: الدكتور عمر محمد اعدوس).

هذه الرسالة تقدم مجموعة متنوعة من التعديلات على مقدر النواة التقليدي المستخدم في طريقة معاينة الخطوط العرضية. وقد تم استخدام تقنية المحاكاة لمقارنة أداء هذه المقدرات بالنسبة لمقدر النواة الحالي مما يساعدنا في البحث عن المقدرات التي لها اداء جيد والتي من المحتمل ان يكون لها مستقبل واعد في طريقة معاينة الخطوط العرضي ة. وعلاوة على ذلك ، فإننا استخدمنا نتائج المقدرات في تحسين مقدر النواة الحالي وذلك بتقليل التحيز السلبي الناتج منه. وقد تناولنا في هذه الرسالة بعض التطبيقات على هذه المقدرات باستخدام مجموعة بيانات حقيقية وناقشناها.

الكلمات المفتاحية: طريقة النواة ، طريقة الخطوط العرضي ة ، التقادير غير المعلمية ، عرض النطاق ، المحاكاة.

CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

Many of the wildlife population studies require an estimation of the population density (abundance). The population density can be estimated by several techniques, one of the most important and efficient technique is the line transect sampling. The populations in line transect sampling might be animate or inanimate items such as animals, birds, trees ... etc. (Burnham et. al., 1980).

In line transect method, an observer attempts to estimate the population density D (abundance) by moving across study area following a line with length L. The observer looking to the right and to the left of the line and it is not sufficient just to record the number of observed objects, n. Instead, the observer must take the perpendicular distance (x) from the line to a detected object as illustrated in Figure (1.1). The fundamental property and advantage of line transect sampling is that not all objects in the study area must be detected, some objects will be missed. Figure (1.2) shows the detected and missed objects together with the perpendicular distances of the detected objects. Moreover, the logical assumption is that objects near the transect line has a grater probability to be detected than objects far from the line.

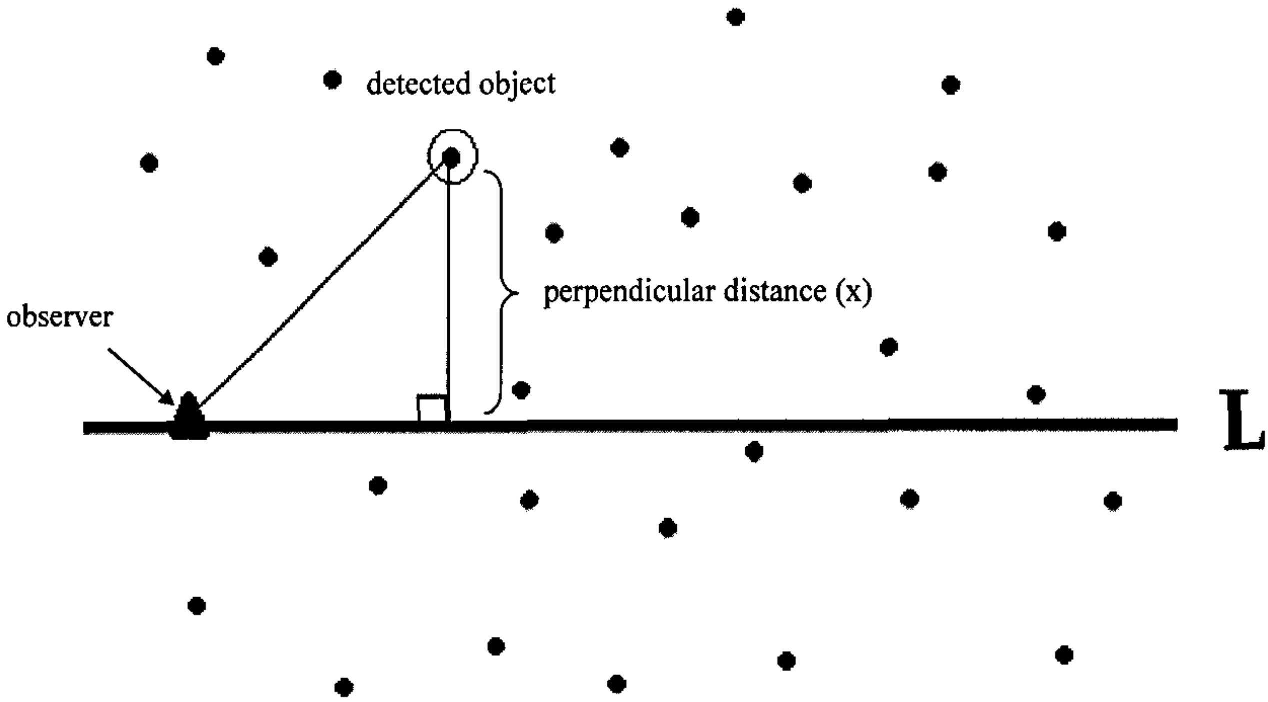


Figure (1.1): The perpendicular distance (x) from the line to a detected object.

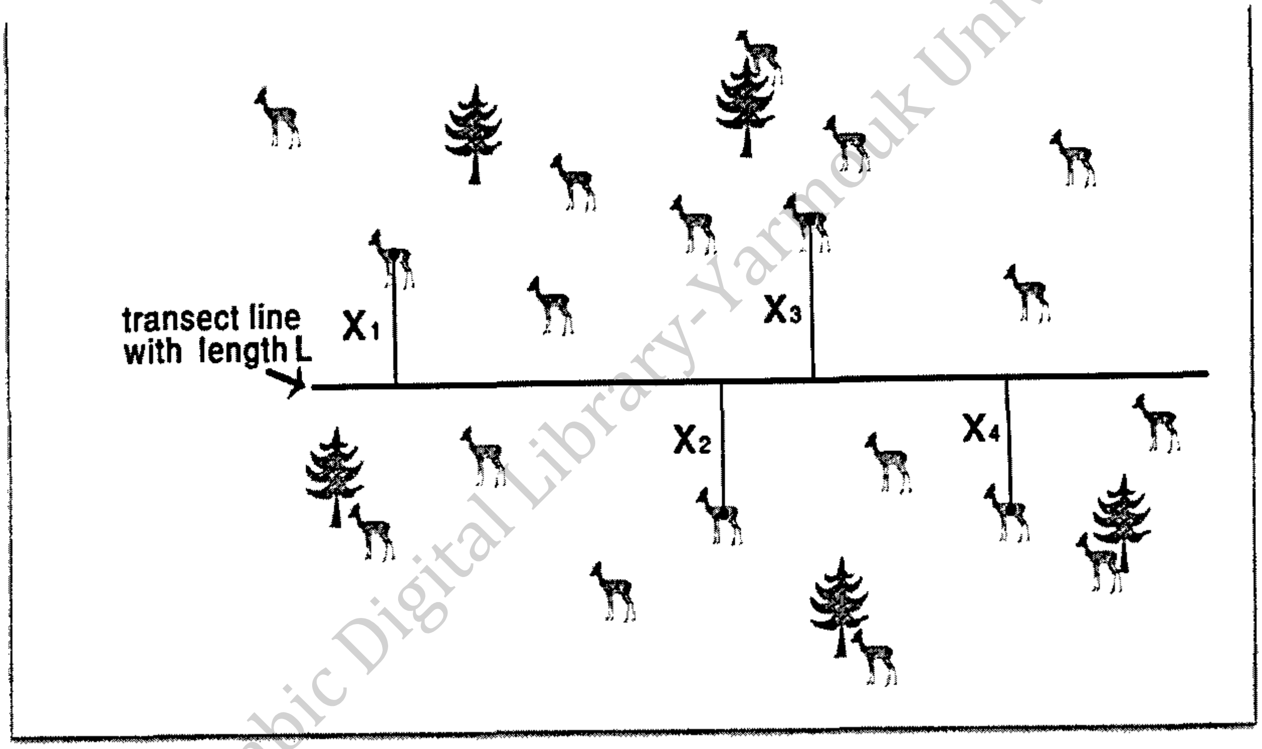


Figure (1.2): The missed objects are represented by , whereas the observed objects are represented by with perpendicular distances $x_1, x_2, x_3,$ and x_4

Assume that the detection function is $g(x)$, that is, $g(x)$ is the conditional probability of observing an object given that the object is at perpendicular distance x from the line. One of the essential assumptions for a reliable estimation of abundance by using line transect sampling is that objects directly on the transect line will never be missed (i.e. $g(0) = 1$). Another logical constraint on $g(x)$ is that $g(x)$ must be monotonically decreasing with x (i.e. object with perpendicular distance x_1 has a greater probability to be detected than object with perpendicular distance x_2 when $x_1 < x_2$).

Let $x_1, x_2, \dots x_n$ be a random sample of size n perpendicular distances that follow the pdf $f(x)$, and let D be the density of objects in a specific study area. Burnham and Anderson (1976) derived a fundamental relationship between $g(x)$, $f(x)$ and D. They showed that $f(x)$ is related to $g(x)$ as

$$f(x) = \frac{g(x)}{\int_0^{\infty} g(x)dx} .$$

That is, $f(x)$ is just $g(x)$ scaled to integrate to 1 and hence,

$$f(0) = \frac{1}{\int_0^{\infty} g(x)dx} ,$$

because $g(0) = 1$. In addition, they showed that the population density D satisfies the following relationship:

$$\mathrm{D} = \frac{\mathrm{E}(n)\, f(0)}{2\,\mathrm{L}} , \qquad (1.1)$$

where n is the number of detected objects, $\mathrm{E}(n)$ is the expected value of n and L is the length of the transect line. The estimator of D is (Burnham et. al., 1980)

$$\hat{\mathrm{D}} = \frac{n\, \hat{f}(0)}{2\,\mathrm{L}} , \qquad (1.2)$$

where $\hat{f}(0)$ is an estimate of the underlying probability density function of perpendicular distances evaluated on the transect line (i.e. $x = 0$).

Thus, in order to estimate D, the crucial problem in line transect sampling is to estimate $f(0)$ as equation in (1.2). In addition, when the estimator of D is obtained then one can obtain the estimator of the population size N, which is given by:

$$\hat{N} = A\hat{D},$$

where A is the population area.

With few exceptions, two separate approaches have been proposed and developed to estimate $f(0)$ in the literature. The first one is the parametric method and the second one is the nonparametric method. The parametric method assumes a functional form of the detection function with unknown parameter θ (θ may be a vector) and then an efficient method of estimation such as the maximum likelihood technique can be used to estimate θ.

The most popular parametric functions that used to model the detection function $g(x)$ are:

(1) The exponential detection function with the form (Gates et. al., 1968)

$$g(x) = e^{-x/\theta} \quad ,x \geq 0, \theta \geq 0$$

which indicates $f(0) = \frac{1}{\theta}$ and the MLE of $f(0)$ is $\hat{f}(0) = \frac{1}{\bar{x}}$, where $\bar{x}$ is the mean of perpendicular distances $x_1, x_2, \dots x_n$.

(2) The half-normal detection function with the form (Hemingway, 1971)

$$g(x) = e^{-x^2/2\sigma^2} \quad ,x \geq 0, \sigma > 0 \ ,$$

which gives $f(0) = \frac{2}{\sigma\sqrt{2\pi}}$. Then the MLE of $f(0)$ is $\hat{f}(0) = \sqrt{\frac{2n}{\pi \sum x_i^2}}$.

The half-normal detection function satisfies the condition $g'(0) = 0$, while the exponential detection function does not. The condition $g'(0) = 0$ is known in line transect literature as the shoulder condition, which indicates that, detection remains nearly certain at small distances from the line transect center.

(3) The exponential power series detection function with two parameters (Pollock, 1978)

$$g(x) = e^{-(x/\lambda)^{\alpha}} \quad , x \geq 0 \, , \lambda > 0, \alpha > 0 \, ,$$

which gives $f(0) = \frac{1}{\lambda\Gamma(1+\frac{1}{\alpha})}$. The MLE of $f(0)$ does not always exist in a closed form and therefore a numerical method is needed to estimate $f(0)$. The exponential power series model is more flexible than the previous two models. It incorporates the exponential model (if $\alpha = 1$) and the half-normal model (if $\alpha = 2$) as special cases. Other models with two parameters are given by Burnham et. al. (1980) and Buckland (1985).

The parametric approach performs well when the detection function $g(x)$ (and consequently the pdf $f(x)$ of the data) is chosen correctly. Otherwise, the performance of the parametric approach is not satisfactory (Buckland et. al., 2001). Therefore, if the parametric approach doesn't satisfy adequacy, the nonparametric approach is candidated as an alternative approach to estimate $f(0)$.

1.2 KERNEL METHOD

The kernel method provides a nonparametric estimator for $f(0)$, which can be considered as a development of histogram for density analysis. This method smooths the histogram density shape as shown in Figure (1.3). The kernel estimator requires no assumptions about the shape of the detection function. It allows the data at hand to talk about itself and then to determine its appropriate probability density function.

The kernel method was introduced by Fix and Hodges (1951) as a way of freeing discriminate analysis from rigid distributional assumptions. Since then it has been developed and used in several statistical applications. Moreover, we can be found the full description of the kernel method in Silverman (1986).

In the recent years, the applications of kernel method are increasing and are distributing at various categories as: astronomy, ecology, earth sciences, econometrics, medicine, water science, etc. (Mack et. al. ,1999)

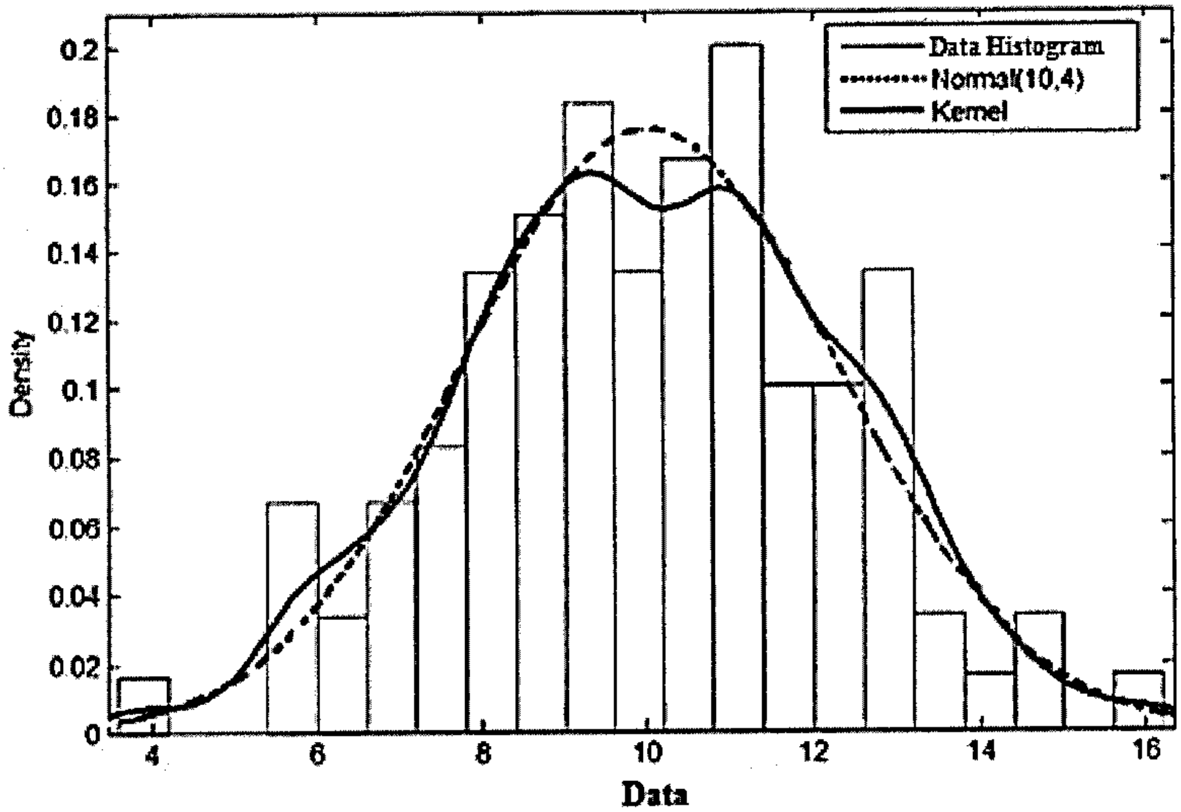


Figure (1.3): An example of nonparametric density estimation. The broken line represents the true population density and the solid line represents the kernel density estimator.

Let $x_1, x_2, \dots x_n$ be a random sample from a continuous pdf $f(x)$, the classical kernel density estimator for $f(x)$ is $\hat{f}(x)$, which is given by (Silverman, 1986),

$$\hat{f}(x) = \frac{1}{n\,h}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right) \quad , -\infty < x < \infty \ , \tag{1.3}$$

where K is a symmetric pdf function called kernel function and h is a positive real number usually called the bandwidth or the smoothing parameter. Under the assumptions that $h \longrightarrow 0$ and $nh \longrightarrow \infty$ when $n \longrightarrow \infty$, Wand and Jones (1995) gave the statistical properties of $\hat{f}(x)$ that contained a small valued terms $o(h^2)$ and $o[(nh)^{-1}]$, and if these small valued terms ignored then the remaining terms known as asymptotic properties.

The expected value of $\hat{f}(x)$ is

$$E(\hat{f}(x)) = f(x) + \frac{1}{2}h^2 f''(x)\int_{-\infty}^{\infty} t^2 K(t)dt + o(h^2) \ ,$$

and then the bias of $\hat{f}(x)$ is

$$bias(\hat{f}(x)) = \frac{1}{2}h^2 f''(x)\int_{-\infty}^{\infty} t^2 K(t)dt + o(h^2) \ ,$$

and the variance of $\hat{f}(x)$ is

$$var(\hat{f}(x)) = \frac{1}{nh} f(x)\int_{-\infty}^{\infty}[K(t)]^2 dt + o[(nh)^{-1}] \ .$$

The asymptotic mean square error (AMSE) of $\hat{f}(x)$ is,

$$AMSE(\hat{f}(x)) = var(\hat{f}(x)) + bias^2(\hat{f}(x)) \ ,$$

$$= \frac{1}{nh} f(x)\int_{-\infty}^{\infty}[K(t)]^2 dt + \frac{1}{4}h^4\left[f''(x)\int_{-\infty}^{\infty} t^2 K(t)dt\right]^2 , \tag{1.4}$$

The first term of the right hand side of (1.4) is the asymptotic variance and the second term is the asymptotic squared bias of $\hat{f}(x)$. As (1.4) shows, the AMSE converge to zero as $h \longrightarrow 0$ and $nh \longrightarrow \infty$ when $n \longrightarrow \infty$.

In line transect sampling the perpendicular distances $x_1, x_2, \dots x_n$ are non-negative. Therefore, we need the kernel estimator $\hat{f}(x)$ of $f(x)$ being defined for non-negative range.

If $0 \le x < \infty$, the kernel estimator for $f(x)$ is (Chen, 1996a),

$$\hat{f}(x) = \frac{1}{n\,h} \sum_{i=1}^{n} \left(K\left(\frac{x-x_i}{h}\right) + K\left(\frac{x+x_i}{h}\right) \right) \quad , 0 \le x < \infty \ .$$

Consequently, the kernel estimator of $f(0)$ is,

$$\hat{f}(0) = \frac{2}{n\,h} \sum_{i=1}^{n} K\left(\frac{x_i}{h}\right) \ . \tag{1.5}$$

Note that, K is a symmetric function, which indicates that $K\left(\frac{-x_i}{h}\right) = K\left(\frac{x_i}{h}\right)$.

Under the assumption that the shoulder condition is true (i.e. $f'(0) = 0$), the bias and the variance of estimator (1.5) are given by

$$bias\left(\hat{f}(0)\right) = h^2 f''(0) \int_0^{\infty} t^2 K(t)\, dt + o(h^2) \ , \tag{1.6}$$

and

$$var\left(\hat{f}(0)\right) = \frac{4f(0)}{nh} \int_0^{\infty} [K(t)]^2\, dt + o[(nh)^{-1}]. \tag{1.7}$$

It becomes well known that the performance of the kernel estimator is very sensitive to the choice of the bandwidth h, while it is not for the choice of kernel function K (Silverman, 1986). In addition, the estimator (1.5) produces an estimate with a large negative bias.

1.2.1 BANDWIDTH SELECTION

The determination value of the bandwidth h has an important impact on the performance of the kernel estimator, which can be selected by using several approaches. One of the most common methods in kernel density estimation is to find h that minimizes the AMSE, which compromises between the variance and bias of $\hat{f}(0)$.

The asymptotic MSE of $\hat{f}(0)$ can be obtained based on (1.6) and (1.7), which is given by,

$$\text{AMSE}\left(\hat{f}(0)\right) = \frac{4f(0)}{nh} \int_0^{\infty} [K(t)]^2\, dt + h^4 \left[f''(0) \int_0^{\infty} t^2 K(t)\, dt\right]^2 . \tag{1.8}$$

By considering equation (1.8) as a function of h (say, T(h)) then differentiate T(h) with respect to h and then equating the result by zero, this yields,

$$h = \left(\frac{f(0) \int_0^\infty [K(t)]^2 \, dt}{\left[f''(0) \int_0^\infty t^2 K(t) \, dt \right]^2} \right)^{\frac{1}{5}} n^{-1/5} \quad . \tag{1.9}$$

The equation (1.9) is somewhat disappointing since it shows that h itself depends on the unknown $f(0)$ being estimated. However, it shows that $h \longrightarrow 0$ and $nh \longrightarrow \infty$ as $n \longrightarrow \infty$. The rate of convergence for the AMSE of $\hat{f}(0)$ can be obtained by substituting the value of h (from (1.9)) back into (1.8). This yields,

$$AMSE\left(\hat{f}(0)\right) = 5n^{-\frac{4}{5}}\left[f(0) \int_0^\infty [K(t)]^2 \, dt\right]^{\frac{4}{5}}\left[f''(0) \int_0^\infty t^2 K(t) \, dt\right]^{\frac{2}{5}}. \tag{1.10}$$

Different methods to obtain the value of h in practice are studied and compared by Gerard and Schucany (1999), who recommended to use the half-normal function as a reference to compute h. Accordingly, in Section (2.4) we used different bandwidth values to investigate their effect on the proposed estimators of $f(0)$.

1.2.2 KERNEL FUNCTION

The selection of kernel function K is discussed in Silverman (1986). He reported that all functions $K(t)$ that satisfy $\int_{-\infty}^{\infty} K(t)dt = 1$, $\int_{-\infty}^{\infty} t\, K(t)dt = 0$ & $\int_{-\infty}^{\infty} t^2\, K(t)dt \neq 0$ which perform equally good results in estimating $f(x)$. The following functions satisfy the above constraints and therefore they can be used as kernel functions (see Silverman, 1986):

(a) $K(t) = \frac{1}{2}$, $|t| < 1$, (Rectangular kernel)

(b) $K(t) = \frac{3}{4}\left(1 - \frac{1}{5}t^2\right)/\sqrt{5}$, $|t| < \sqrt{5}$, (Epanechnikov kernel)

(c) $K(t) = \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}$, $-\infty < t < \infty$, (Gaussian kernel).

Usually, the kernel function K is chosen to be a unimodal and symmetric density to yield unbiased estimates by using a symmetric distribution of the weights on both sides of the point of estimate. However, the kernel functions that do not satisfy these requirements are inadmissible (see Cline, 1988). In this study, the kernel function K is chosen to be the Gaussian kernel unless otherwise is stated. Assume that K is the Gaussian kernel (i.e. $K = N(0,1)$) and $f(x)$ is the half-normal with scale parameter σ^2 then the optimal value of h based on equation (1.9) to estimate $f(0)$ is

$$h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}}. \qquad (1.11)$$

where $\hat{\sigma}$ is the maximum likelihood estimator of σ, which is given by $\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}$.

The formula (1.11) is recommended by Gerard and Schucany (1999).

1.3 LITERATURE REVIEW

The line transect sampling comprehensive reviews can be found in Seber (1982) and Buckland et. al. (2001). Quang (1990) presented a method of deriving an approximate confidence interval for the abundance D. Chen (1996a and 1996b) estimated the density (abundance) of population based on explicitly modeling the probability density function of the perpendicular sighting distances without any assumptions on the form of the detection function. He proposed the classical kernel method to estimate D. Buckland (1992) proposed a Hermite polynomial model to estimate $f(0)$. Barabesi (2000) used a local likelihood density estimation to reduce the boundary bias. He also proposed a semiparametric kernel estimator. Also, Barabesi et. al. (2002) developed a semiparametric method for grouped data with local least squares to obtain estimates for $f(0)$.

Melville and Welsh (2001) proposed an approach to line transect sampling using a separate calibration study to estimate the detection function. Marques (2004) investigated the effect of error measurement on line transect estimators.

Recent years researches have focused on the use of kernel density estimation method to estimate $f(0)$ and consequently the population abundance D. Mack (1998) used the kernel method to test the validity of the shoulder condition assumption. Mack et. al. (1999) proposed kernel estimation in transect sampling without the shoulder condition assumption.

Gerard and Schucany (2002) suggested a method to combine kernel estimators from individuals transects when each transect has sufficient data to support estimation. Mack (2002) investigated some methods to correct the bias when kernel method is used in constructing confidence intervals for population abundance. Eidous (2005) investigated some possibilities for improvements of kernel density estimates using line transect sampling and he proposed two nonparametric estimators.

Fewster et. al. (2005) considered line transect sampling in small and large regions. There are some other methods for line transect sampling and some applications, such as, Hiby and Krishna (2001) who studied line transect sampling from a curving path. Okamura (2003) proposed line transect method to estimate abundance of long-diving animals. Marshall et. al. (2008) considered the problem of selection of line transect methods for estimating the density of group-living in Primatology. Gong et. al. (2010) have construct confidence intervals based on kernel estimation with a different stopping rule. Eidous (2011) suggested the variable location kernel method to estimate $f(0)$ using line transect technique.

Eidous and Shakhatreh (2011) proposed a semi-parametric estimator for $f(0)$ when the shoulder condition is assumed to be valid and when it is violated. Their estimator combines between the kernel estimator and a specific parametric estimator.

1.4 RESEARCH OBJECTIVES

This thesis aims to propose and to study the performances of new adaptations of kernel estimators. The proposed estimators are compared with the classical kernel estimator $\hat{f}(0)$, which is introduced by equation (1.5). Also, we study and compare the performance of the new estimators with different values of the bandwidth h and the parameter p (will be discussed in Chapter II).

Moreover, we will try to correct the negative bias that associated the classical kernel estimator $\hat{f}(0)$ by multiplying this estimator with some correction factors (will be discussed in Chapter II).

At the end, we will study the different proposed estimators for two real data sets.

1.5 THESIS OUTLINE

This thesis is organized in the following manner. Chapter I covers vital concepts about line transect sampling and the traditional kernel estimator for $f(0)$. In addition, some existing methods to estimate $f(0)$ are introduced in this chapter.

Chapter II introduces the main results of this thesis. The different proposed estimators are given in this chapter together with their simulation results. An important theorem related the proposed estimators is stated and proved in this chapter. In chapter III, we use the proposed estimators and the classical kernel estimator to estimate the population abundance for two real data sets. And in chapter IV, we suggest some conclusions and implement future research.

CHAPTER TWO

ESTIMATION OF $f(0)$

2.1 INTRODUCTION

In this chapter we introduced a nonparametric estimator for the parameter $f(0)$ using line transect sampling. The proposed estimator is a general form that considers the classical kernel estimator as a special case. Some mathematical properties related to the proposed estimator are given in this chapter and the simulation technique is used to investigate the performance of this new estimator. The effect of the choice of the bandwidth h, the sample size n, and the choice of the detection function model is also addressed.

2.2 PROPOSED KERNEL ESTIMATOR

The classical kernel estimator using line transect sampling gives a poor performance in some cases with a large negative bias (see Quang, 1993 and Eidous, 2005). In this thesis we are trying to adaptive this estimator. In addition, we will refer to the estimators that give poor performance –with respect to the classical kernel estimator $\hat{f}(0)$– so as to exclude them in future research.

A deep insight into equation (1.5) indicates that the classical kernel estimator $\hat{f}(0)$ is the mean of the random variables $Y_1, Y_2, \dots, Y_n$, where $Y_1 = \frac{2}{h} K\left(\frac{x_1}{h}\right)$, $Y_2 = \frac{2}{h} K\left(\frac{x_2}{h}\right)$, … , $Y_n = \frac{2}{h} K\left(\frac{x_n}{h}\right)$.

Therefore, estimator (1.5) can be re-written as the following:

$$\hat{f}(0) = \frac{1}{n} \sum_{i=1}^{n} Y_i \quad . \qquad (2.1)$$

Because the estimator in (2.1) is a measure of center (mean) for the values $Y_1, Y_2, \dots, Y_n$, we propose to consider the general measure of center estimator, which is given by

$$\hat{f}_p(0) = \left(\frac{1}{n}\sum_{i=1}^{n} Y_i^{\,p}\right)^{\frac{1}{p}} \qquad (2.2)$$

where $Y_i = \frac{2}{h} K\left(\frac{x_i}{h}\right)$, $i = 1,2,..,n$ and $p \in \mathbb{R}$. Now, the estimator (2.1) is a special case of estimator (2.2) when $p = 1$. The main central idea in this thesis is to study the statistical properties of estimator (2.2) for different values of p. In addition, a new estimator that combines the classical kernel estimator, $\hat{f}(0)$ and estimator (2.2) is proposed to reduce the negative bias that is associated $\hat{f}(0)$ and its statistical properties are also studied.

As special cases of $\hat{f}_p(0)$, we will consider –in particular– the following five estimators:

(1) For $p = 1$ then (2.2) reduces to (2.1). We denoted the resultant estimator by $\hat{f}_1(0)$ and call it the mean kernel estimator.

(2) For $p = -1$, we obtain,

$$\hat{f}_{-1}(0) = \left(\frac{1}{n}\sum_{i=1}^{n} Y_i^{-1}\right)^{-1} = \frac{n}{\frac{1}{Y_1}+\frac{1}{Y_2}+\cdots+\frac{1}{Y_n}} = \frac{n}{\frac{h}{2}\sum_{i=1}^{n}\frac{1}{K\left(\frac{x_i}{h}\right)}}.$$

$\hat{f}_{-1}(0)$ is the harmonic mean of $Y_1, Y_2, \dots, Y_n$. We call $\hat{f}_{-1}(0)$ the harmonic kernel estimator of $f(0)$.

(3) As $p \to 0$, we obtain the geometric kernel estimator, which is given by

$$\lim_{p\to 0} \hat{f}_p(0) = (Y_1 Y_2 \dots Y_n)^{\frac{1}{n}}$$

$$= \left(\frac{2}{h} K\left(\frac{x_1}{h}\right) \cdot \frac{2}{h} K\left(\frac{x_2}{h}\right) \dots \frac{2}{h} K\left(\frac{x_n}{h}\right)\right)^{\frac{1}{n}}.$$

For simplicity, we denote the above estimator by $\hat{f}_0(0)$. The proof of the above result is given below.

Proof:

$$\hat{f}_p(0) = \left(\frac{1}{n}\sum_{i=1}^{n} Y_i{}^p\right)^{\frac{1}{p}}$$

$$= \frac{\left(\sum_{i=1}^{n} Y_i{}^p\right)^{\frac{1}{p}}}{(n)^{\frac{1}{p}}}.$$

By taking the natural logarithm of both sides, we obtain,

$$ln\,\hat{f}_p(0) = \frac{1}{p} ln\left(\sum_{i=1}^{n} Y_i{}^p\right) - \frac{1}{p} ln\, n$$

$$= \frac{ln\left(\sum_{i=1}^{n} Y_i{}^p\right) - ln\, n}{p}.$$

Therefore,

$$\lim_{p\to 0} ln\,\hat{f}_p(0) = \lim_{p\to 0} \frac{ln\left(\sum_{i=1}^{n} Y_i{}^p\right) - ln\, n}{p}.$$

By using L'Hôpital's rule we obtain,

$$\lim_{p\to 0} ln\,\hat{f}_p(0) = \lim_{p\to 0} \frac{ln\left(\sum_{i=1}^{n} Y_i{}^p\right) - ln\, n}{p}$$

$$= \lim_{p\to 0} \frac{\frac{\sum_{i=1}^{n} Y_i{}^p\, ln(Y_i) - 0}{\sum_{i=1}^{n} Y_i{}^p}}{1}$$

$$= \lim_{p\to 0} \frac{\sum_{i=1}^{n} Y_i{}^p\, ln(Y_i)}{\sum_{i=1}^{n} Y_i{}^p}$$

$$= \frac{\sum_{i=1}^{n} ln(Y_i)}{\sum_{i=1}^{n} 1}$$

$$= \frac{1}{n} ln(Y_1.Y_2 \ldots Y_n)$$

$$\lim_{p\to 0} \ln \hat{f}_p(0) = \ln(Y_1.Y_2 \dots Y_n)^{\frac{1}{n}} .$$

Because $\lim_{p\to 0} \ln \hat{f}_p(0) = \ln(Y_1.Y_2 \dots Y_n)^{\frac{1}{n}}$, then

$$\lim_{p\to 0} \hat{f}_p(0) = e^{\ln(Y_1.Y_2 \dots Y_n)^{\frac{1}{n}}} = (Y_1.Y_2 \dots Y_n)^{\frac{1}{n}} .$$

Thus, as $p \to 0$

$$\hat{f}_0(0) = (Y_1 Y_2 \dots Y_n)^{\frac{1}{n}}$$

$$= \left(\frac{2}{h} K\left(\frac{x_1}{h}\right) . \frac{2}{h} K\left(\frac{x_2}{h}\right) \dots \frac{2}{h} K\left(\frac{x_n}{h}\right)\right)^{\frac{1}{n}} = \frac{2}{h}\left(\prod_{i=1}^{n} K\left(\frac{x_i}{h}\right)\right)^{1/n} .$$

(4) For $p = 1.5$, we obtain,

$$\hat{f}_{1.5}(0) = \left(\frac{1}{n}\sum_{i=1}^{n} Y_i^{\frac{3}{2}}\right)^{\frac{2}{3}}$$

$$= \left(\frac{Y_i^{3/2} + Y_i^{3/2} + \dots + Y_i^{3/2}}{n}\right)^{\frac{2}{3}}$$

$$= \left(\frac{\left(\frac{2}{h} K\left(\frac{x_1}{h}\right)\right)^{3/2} + \left(\frac{2}{h} K\left(\frac{x_2}{h}\right)\right)^{3/2} + \dots + \left(\frac{2}{h} K\left(\frac{x_n}{h}\right)\right)^{3/2}}{n}\right)^{\frac{2}{3}} .$$

(5) For $p = 2$, we obtain the quadratic kernel estimator,

$$\hat{f}_2(0) = \left(\frac{1}{n}\sum_{i=1}^{n} Y_i^{2}\right)^{\frac{1}{2}}$$

$$= \sqrt{\frac{Y_1^{\,2} + Y_2^{\,2} + \dots + Y_n^{\,2}}{n}}$$

$$\hat{f}_2(0) = \sqrt{\frac{\left(\frac{2}{h} K\left(\frac{x_1}{h}\right)\right)^2 + \left(\frac{2}{h} K\left(\frac{x_2}{h}\right)\right)^2 + \cdots + \left(\frac{2}{h} K\left(\frac{x_n}{h}\right)\right)^2}{n}}.$$

We note here that there are many other possible values for p which can be selected from the range of p where $p \in \mathbb{R}$.

2.3 PRELIMINARY SIMULATION RESULTS

A preliminary simulation study is performed to investigate the performances of $\hat{f}_p(0)$ for $p = -1, 0, 1, 1.5, 2$. In addition, some other estimators when $p < -1$ and when $p > 2$ are also investigated.

In this simulation study, we selected *K(x)* to be Gaussian kernel (i.e. $K = N(0,1)$) (see Subsection 1.2.2) for all estimators and the smoothing parameter h is computed by using the equation $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}}$, also for all estimators, where $\hat{\sigma}$ is the MLE of σ (with reference to half-normal model) (Chen, 1996a and Gerard and Schucany, 1999).

The interested proposed estimators are implemented using data that are simulated from 12 detection functions. These 12 detection functions are selected based on three families of models, which are commonly used as references in line transect studies (see Barabesi, 2001 and Eidous, 2005).

In this simulation study, the 12 detection functions were truncated at distance w, where the truncation point w in line transect sampling is defined to be the maximum perpendicular distance at which the distances beyond this value are ignored.

The three families are:

© Arabic Digital Library-Yarmouk University

a) The exponential power (EP) family (Pollock, 1978)

$$f(x) = \frac{1}{\Gamma(1+1/\beta)} e^{-x^{\beta}} \quad , \qquad x \geq 0 , \beta \geq 1 ,$$

with detection function $g(x) = e^{-x^{\beta}}$. Four models were selected from this family with parameter values $\beta = 1.0, 1.5, 2.0,$ and 2.5 and corresponding truncation points $w = 5.0$, $3.0, 2.5$ and 2.0 (see Figure 2.1).

b) The hazard-rate (HR) family (Hayes and Buckland, 1983)

$$f(x) = \frac{1}{\Gamma(1-1/\beta)} \left(1 - e^{-x^{-\beta}}\right) \quad , \qquad x \geq 0 , \beta > 1 ,$$

with detection function $g(x) = 1 - e^{-x^{-\beta}}$. Four models were selected from this family with parameter values $\beta = 1.5, 2.0, 2.5,$ and 3.0 and corresponding truncation points $w = 20.0$, $12.0, 8.0,$ and 6.0 (see Figure 2.2).

c) The beta (BE) model (Eberhardt, 1968)

$$f(x) = (1+\beta)(1-x)^{\beta} \quad , \qquad 0 \leq x < 1 , \beta \geq 0 ,$$

with detection function $g(x) = (1-x)^{\beta}$. We selected four models from this model with parameter values $\beta = 1.5, 2.0, 2.5,$ and 3.0 and $w = 1.0$ for all cases (see Figure 2.3).

These 12 detection functions cover a wide range of possible models for perpendicular distances in practice, which vary near the perpendicular distance $x = 0$ from spike to flat. The spike property occurs for models with $f'(0) \neq 0$ (e.g. EP model with $\beta = 1$ and BE model with all values of β), while the flat property (known in line transect literature as shoulder condition) occurs for models with $f'(0) = 0$ (e.g. EP model with $\beta = 1.5, 2.0, 2.5$ and HR model with different values of β).The simulation results are based on simulated 1000 samples of size $n = 50, 100, 200$. For each estimator, we compute the relative bias RB,

$$RB = \frac{E\left(\hat{f}(0)\right) - f(0)}{f(0)} \quad ,$$

and the relative mean error RME,

$$RME = \frac{\sqrt{MSE\left(\hat{f}(0)\right)}}{f(0)} .$$

The important results of the preliminary simulation study were:

a. The estimator $\hat{f}_p(0)$ is very sensitive to the choice of p (i.e. the variation of p is a critical point in the estimation performances using RB and RME).

b. The performances of $\hat{f}_0(0)$ and $\hat{f}_{-1}(0)$ are very bad –with respect to the classical kernel estimator $\hat{f}(0)$ – (see Table 2.10).

c. Fixed the value of p, the performance of $\hat{f}_p(0)$ can be improved by changing the value of h for some values of p $(1 \leq p \leq 2)$. However, for many values of p, the performances of $\hat{f}_p(0)$ cannot be improved for any values of h, specially when p takes values near zero or less, and when p takes values greater than two.

d. The performances of $\hat{f}_p(0)$ are satisfactory for $1 \leq p \leq 2$. The performance of $\hat{f}_p(0)$ becomes very poor as we take p away below 1 or away above 2.

Therefore, the two estimators $\hat{f}_{-1}(0)$ and $\hat{f}_0(0)$ are excluded in this study. Instead of them, we will consider the estimator $\hat{f}_p(0)$ with $p = 1.1$ and $p = 1.3$, which are given below:

$$\hat{f}_{1.1}(0) = \left(\frac{1}{n}\sum_{i=1}^{n} Y_i^{1.1}\right)^{1/1.1} ,$$

and

$$\hat{f}_{1.3}(0) = \left(\frac{1}{n}\sum_{i=1}^{n} Y_i^{1.3}\right)^{1/1.3} .$$

Many other possible values for p that fall in the interval [1,2] cab be chosen. However, we restrict our selection for the values p =1.0, 1.1, 1.3, 1.5 and 2.0. We expect that these values give us a good idea about the performances of $\hat{f}_p(0)$ for all $p \in [1,2]$.

Before we are going throughout the studying of the practical implementation of the above estimators we state the following important theorem.

Theorem (2.1):

Let $\hat{f}_p(0) = \left(\frac{1}{n} \sum_{i=1}^{n} Y_i^{\,p}\right)^{1/p}$ and let $Y_i = \frac{2}{h} K\left(\frac{x_i}{h}\right)$, $i = 1,2,\dots,n$, then for a fixed value of h,

(1) $\lim_{p\to\infty} \hat{f}_p(0) = \max\{Y_1, Y_2, \dots, Y_n\}$

(2) $\lim_{p\to-\infty} \hat{f}_p(0) = \min\{Y_1, Y_2, \dots, Y_n\}$

(3) $\hat{f}_p(0)$ is non-decreasing function in p.

Proof of Theorem (2.1)

(1) Let $Y_{(n)}$ be the maximum value of $Y_1, Y_2, \dots, Y_n$. That is

$$Y_{(n)} = \max\{Y_1, Y_2, \dots, Y_n\}.$$

Now,

$$\ln \hat{f}_p(0) = \frac{1}{p}\left[\ln\left(\sum_{i=1}^{n} Y_i^p\right) - \ln n\right]$$

$$= \frac{1}{p}\left[\ln\left[Y_i^p\left(\left(\frac{Y_1}{Y_{(n)}}\right)^p + \left(\frac{Y_2}{Y_{(n)}}\right)^p + \dots + \left(\frac{Y_n}{Y_{(n)}}\right)^p\right)\right] - \ln n\right]$$

$$= \frac{1}{p}\left[p \ln Y_{(n)} + \ln\left(\left(\frac{Y_1}{Y_{(n)}}\right)^p + \left(\frac{Y_2}{Y_{(n)}}\right)^p + \dots + \left(\frac{Y_n}{Y_{(n)}}\right)^p\right) - \ln n\right]$$

$$= \ln Y_{(n)} + \frac{\ln\left(\left(\frac{Y_1}{Y_{(n)}}\right)^p + \left(\frac{Y_2}{Y_{(n)}}\right)^p + \dots + \left(\frac{Y_n}{Y_{(n)}}\right)^p\right)}{p} - \frac{\ln n}{p}$$

Now,

$$\lim_{p\to\infty} \ln \hat{f}_p(0) = \ln Y_{(n)} + \lim_{p\to\infty}\left[\frac{\ln\left(\left(\frac{Y_1}{Y_{(n)}}\right)^p + \left(\frac{Y_2}{Y_{(n)}}\right)^p + \cdots + \left(\frac{Y_n}{Y_{(n)}}\right)^p\right)}{p}\right] - 0.$$

By using L'Hôpital's rule for the second term of the right hand side we obtain,

$$\lim_{p\to\infty}\left[\frac{\left(\frac{Y_1}{Y_{(n)}}\right)^p \ln\left(\left(\frac{Y_1}{Y_{(n)}}\right)\right) + \left(\frac{Y_2}{Y_{(n)}}\right)^p \ln\left(\left(\frac{Y_2}{Y_{(n)}}\right)\right) + \cdots + \left(\frac{Y_n}{Y_{(n)}}\right)^p \ln\left(\left(\frac{Y_n}{Y_{(n)}}\right)\right)}{\left(\frac{Y_1}{Y_{(n)}}\right)^p + \left(\frac{Y_2}{Y_{(n)}}\right)^p + \cdots + \left(\frac{Y_n}{Y_{(n)}}\right)^p}\right]$$

Since the term $Y_{(n)}$ equals at least one of the values $Y_1, Y_2, \ldots, Y_n$ then at least one term in the denominator equals one and the other approach to zero when $p \to \infty$. Also, at least one term in the numerator equals $\ln(1) = 0$ and the others approach to zero when $p \to \infty$. That is, all terms in the numerator are zero when $p \to \infty$. Therefore, the limiting is zero and then

$$\lim_{p\to\infty} \ln \hat{f}_p(0) = \ln Y_{(n)} \ ,$$

which implies,

$$\hat{f}_p(0) = Y_{(n)}$$
$$= \max\{Y_1, Y_2, \ldots, Y_n\}.$$

(2) Let $Y_{(1)}$ be the minimum value of $Y_1, Y_2, \ldots, Y_n$. That is

$$Y_{(1)} = \min\{Y_1, Y_2, \ldots, Y_n\}.$$

Same as (1) but we take $Y_{(1)}$ as coefficient of the terms $Y_1 + Y_2 + \cdots + Y_n$ as $\left(\frac{Y_i}{Y_{(1)}}\right)$. Then

$$\hat{f}_p(0) = Y_{(1)}$$
$$= \min\{Y_1, Y_2, \ldots, Y_n\}.$$

(3) We need to prove that $\hat{f}_p(0)$ is a non-decreasing function in p.

Let $a = Y_i$, $b = Y_{i+1}$ and $x = p$, where $a, b > 0$ and $x \in (-\infty, \infty)$

then

$$\hat{f}_p(0) \equiv f(x) = \left(\frac{a^x + b^x}{2}\right)^{1/x}$$

$$\ln f(x) = \frac{1}{x}[\ln(a^x + b^x) - \ln 2]$$

$$= \frac{1}{x}\left[\ln\left[a^x\left(1 + \left(\frac{b}{a}\right)^x\right)\right] - \ln 2\right]$$

$$= \frac{1}{x}\left[x \ln a + \ln\left(1 + \left(\frac{b}{a}\right)^x\right) - \ln 2\right]$$

$$= \ln a + \frac{\ln\left(1 + \left(\frac{b}{a}\right)^x\right) - \ln 2}{x} .$$

Now,

$$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$$

$$= \frac{x\left[\frac{\left(\frac{b}{a}\right)^x \ln\left(\frac{b}{a}\right)}{1 + \left(\frac{b}{a}\right)^x}\right] - \ln\left(1 + \left(\frac{b}{a}\right)^x\right) + \ln 2}{x^2} .$$

Therefore,

$$f'(x) = \left(\frac{x\left[\frac{\left(\frac{b}{a}\right)^x \ln\left(\frac{b}{a}\right)}{1 + \left(\frac{b}{a}\right)^x}\right] - \ln\left(1 + \left(\frac{b}{a}\right)^x\right) + \ln 2}{x^2}\right) * f(x).$$

To show that $f(x)$ is non-decreasing in x, it is enough to show that $f'(x) \geq 0, \forall x$.

Because $f(x) > 0$, $\forall x$ then from the last expression,

$$f'(x) \geq 0 \text{ iff } x\left[\frac{\left(\frac{b}{a}\right)^x \ln\left(\frac{b}{a}\right)}{1+\left(\frac{b}{a}\right)^x}\right] - \ln\left(1+\left(\frac{b}{a}\right)^x\right) + \ln 2 \geq 0$$

$$\text{iff } x\left[\frac{\left(\frac{b}{a}\right)^x \ln\left(\frac{b}{a}\right)}{1+\left(\frac{b}{a}\right)^x}\right] \geq \ln\left(\frac{1+\left(\frac{b}{a}\right)^x}{2}\right)$$

$$\text{iff } \ln(D)^{xD^x/(1+D^x)} \geq \ln\left(\frac{1+D^x}{2}\right), \quad \text{where } D = \frac{b}{a} > 0$$

$$\text{iff } D^{\frac{xD^x}{1+D^x}} \geq \frac{1+D^x}{2}$$

$$\text{iff } D^{xD^x} \geq \frac{(1+D^x)^{1+D^x}}{2^{1+D^x}}$$

$$\text{iff } 2^{1+D^x} D^{xD^x} \geq (1+D^x)^{1+D^x}$$

$$\text{iff } 2^{R+1} R^R \geq (R+1)^{R+1}, \quad \text{where } R = D^x \quad, \quad R > 0$$

$$\text{iff } R^R \geq \left(\frac{R+1}{2}\right)^{R+1}$$

$$\text{iff } \left(\frac{2R}{R+1}\right)^R \frac{2}{R+1} \geq 1 \quad .$$

Consider the left hand side of the last expression as a function of R (say $g(R)$) then

$$g(R) = \left(\frac{2R}{R+1}\right)^R \frac{2}{R+1},$$

and

$$\ln g(R) = R[\ln 2 + \ln R - \ln(R+1)] + \ln 2 - \ln(R+1).$$

Now,

$$g'(R) = \left[R\left[\frac{1}{R} - \frac{1}{R+1}\right] + \ln 2 + \ln R - \ln(R+1) - \frac{1}{R+1}\right] g(R)$$

$$= [\ln 2 + \ln R - \ln(R+1)]g(R) \quad .$$

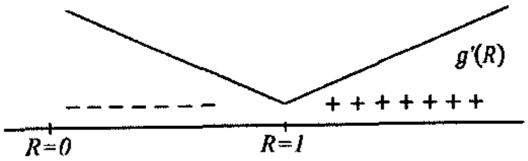
For the last expression, take $g'(R) = 0$ then $\ln 2 + \ln R - \ln(R+1) = 0$ because $g(R) > 0, \forall R > 0$. Therefore,

$\ln\frac{2R}{R+1} = 0$ which implies $e^0 = \frac{2R}{R+1}$ and then $2R = R + 1$. This gives

$$R = 1$$

is the root of $g'(R)$.

Now, check the sign of $g'(R)$ for $0 < R < 1$ and for $R > 1$, you obtain



,

which indicates that $g(R)$ decreases for $R \in (0,1)$ and increases for $R \in (1, \infty)$.

Therefore, $R = 1$ is a minimum point for $g(R)$ and $g(R = 1) = 1$ is a minimum value for $g(R)$. Therefore $g(R) \geq 1, \forall\, R > 0$, which indicates that $f'(x) > 0$. This complete the proof.

Theorem (2.1) shows how the behavior of $\hat{f}_p(0)$ will be when p increases or when p decreases. Of course and based on Theorem (2.1), we expect that the performance of $\hat{f}_p(0)$ will be not satisfactory when p takes large values or when p takes small values. While we are searching for a good measure of center for the values $Y_1, \dots, Y_n$, the above theorem tells us that $\hat{f}_p(0)$ will tend to be the smallest value of $Y_1, \dots, Y_n$ as p tends to $-\infty$ and $\hat{f}_p(0)$ tends to be the largest of $Y_1, \dots, Y_n$ when p tends to ∞. In addition, the above theorem tells us that the value of estimator $\hat{f}_p(0)$ will be increased when p increases. Therefore, at some value of p, the estimator $\hat{f}_p(0)$ becomes overestimate of the true parameter $f(0)$. On the other hand, when p decreases, the estimator $\hat{f}_p(0)$ decreases and at some values of p the estimator becomes underestimate of the true value $f(0)$. However, the determination of the critical value of p when the estimate becomes overestimate or the estimate becomes underestimate can't be determined easily because we are used various models and various bandwidth h.

These results are demonstrated clearly in the tables of simulation results at the end of this chapter.

2.4 OTHER PROPOSED ESTIMATORS

In this section we introduce other kernel estimators for $f(0)$ using line transect sampling to enrich this research.

The motivation of the proposed estimators in this section is due to the negative bias that associated the current classical kernel estimator $\hat{f}_1(0)$. This fact is demonstrated clearly in Table (2.1) at which the biases of $\hat{f}_1(0)$ are negative for all models that are considered. Inspection of Tables (2.3) and (2.5) show that the estimator $\hat{f}_1(0)$ produces a negative bias, even when the value of the smoothing parameter h is slightly decreased or increased. This means that the classical kernel method produces underestimate values for the true parameter $f(0)$. This result is also addressed in the work of Chen (1996a), Gerard and Schucany (1999) and Eidous (2005). To correct this problem, we proposed another set of estimators in this section. The proposed estimator combines of the classical kernel estimator and the estimator (2.2). It simply consists of $\hat{f}_1(0)$ multiply by a factor. (Which is greater than one) used to correct the performance of $\hat{f}_1(0)$. The form of the proposed estimator is,

$$\tilde{f}^*_m(0) = \hat{f}_1(0)\, Exp\left[\frac{\hat{f}_{2m-1}(0)}{\hat{f}_m(0)} - 1\right], \qquad (2.3)$$

where $m \in (1, \infty)$. If $m = 1$ then (2.3) reduces to be $\hat{f}_1(0)$.

Based on Theorem (2.1), $\hat{f}_p(0)$ is a non-decreasing function in p, which indicates that $\hat{f}_{p1}(0) \geq \hat{f}_{p2}(0)$,$\forall$ $p1 > p2$. Because $2m - 1 \geq m$, $\forall$ $m \geq 1$ then $\hat{f}_{2m-1}(0) \geq \hat{f}_m(0)$ and $\frac{\hat{f}_{p2}(0)}{\hat{f}_{p1}(0)} - 1 \geq 0$. Therefore, the factor $Exp\left[\frac{\hat{f}_{2m-1}(0)}{\hat{f}_m(0)} - 1\right]$ is always greater than or equal to one. This indicates that $\tilde{f}_m^*(0) \geq \hat{f}_1(0)$,$\forall\ m \geq 1$, where the equality of the two estimators occurs when $m = 1$.

The question arises now is: why the corrected factor is chosen to be $Exp\left[\frac{f_{2m-1}(0)}{f_m(0)} - 1\right]$? Why it is not $Exp\left[\frac{f_{m^2}(0)}{f_m(0)} - 1\right]$ or $Exp\left[\frac{f_{10m+m^2}(0)}{f_{m^2}(0)} - 1\right]$ or ... etc? Or even, why it is not $Exp\left[\frac{f_{m_1}(0)}{f_{m_2}(0)} - 1\right]$? Where m_1 and m_2 are any two values such that $m_1 > m_2$ to ensure that the exponent term is greater than zero.

The corrected factors that mention in the above questions (or any other suggested factors) may be worked well, but this needs to study them at least via simulation technique to decide about their performances. Actually, our simulation trials lead us to use the corrected factor of the estimator in (2.3), which in turns improves the performances of $\hat{f}_1(0)$ in most cases that are considered as the simulation results of Tables (2.7), (2.8) and (2.9) demonstrated.

However, we introduced other four corrected factors with the form $Exp\left[\frac{f_{m_1}(0)}{f_{m_2}(0)} - 1\right]$, where m_1 and m_2 are selected arbitrary under the constraint $m_1 > m_2$. Based on these arbitrary choices, we proposed the following estimators:

1- $\tilde{f}_a^*(0) = \hat{f}_1(0) * Exp\left[\frac{f_{1.3}(0)}{f_1(0;h,n)} - 1\right]$

2- $\tilde{f}_b^*(0) = \hat{f}_1(0) * Exp\left[\frac{f_2(0)}{f_{1.5}(0)} - 1\right]$

3- $\tilde{f}_c^*(0) = \hat{f}_1(0) * Exp\left[\frac{f_{20}(0)}{f_{10}(0)} - 1\right]$, and

4- $\tilde{f}_d^*(0) = \hat{f}_1(0) * Exp\left[\frac{f_{50}(0)}{f_{20}(0)} - 1\right]$.

As we mentioned before, there are many other choices for m_1 and m_2 to produce new estimators for $f(0)$.

2.5 SIMULATION DESIGN AND ESTIMATORS

One of the important techniques used to compare the performances between different estimators is the simulation technique. Moreover, the researchers may investigate the relative superiority of each estimator without actually deriving the mathematical properties.

In this section, a simulation study is conducted to compare between the performances –using RB and RME– of the different proposed estimators. The classical kernel estimator $\hat{f}(0)$ is also considered and the performances of this estimator are considered to be a basis of comparison.

More interpretative, in this study, we selected *K(x)* to be Gaussian kernel for all estimators and the smoothing parameter h is computed by using the equation $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}}$, also for all estimators, where $\hat{\sigma}$ is the MLE of σ.

The data are simulated form the 12 models that are given in Section (2.3). Also, the RB and the RME are same as given in Section (2.3).

We keep in mind that the estimator $\hat{f}_p(0)$ is very sensitive to the choice of p and its performance is very bad for $p < 1$ and for $p > 2$ (see section 2.3), this simulation study trying to answer the following questions:

(1) Is the performance of $\hat{f}_p(0)$ acceptable for $1 \leq p \leq 2$?

(2) Can we improve the performance of $\hat{f}_p(0)$ by varying the value of the smoothing parameter h ?

(3) Are the arbitrary estimators $\tilde{f}_a^*(0), \tilde{f}_b^*(0), \tilde{f}_c^*(0)$ and $\tilde{f}_d^*(0)$ give any improvements over the estimater $\hat{f}_p(0)$?

(4) Can we improve the performances of these arbitrary estimators by changing the smoothing parameter h ?

(5) Does the formula of the proposal estimator $\tilde{f}_m^*(0)$ work well to estimate $f(0)$? What is the best value for m ?

To answer Question (1), four values of p are selected, which are $p = 1.1, 1.3, 1.5, 2$, and besides the value $p = 1$ which gives the classical kernel estimator $\hat{f}(0)$, i.e. $\hat{f}_1(0) = \hat{f}(0)$. The stimulation results are given in the first five columns of Tables (2.1) and (2.2).

To answer Question (2), the optimal rule $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}}$ (Gerard and Schucany, 1999) for the smoothing parameter h is used then we vary h by multiply it with 1.2 and 0.8. This gives new smoothing parameters; $h_1 = 1.2h$ and $h_2 = 0.8h$. These new values for the smoothing parameter are also considered to answer Question (4).

It is worthwhile to mention here that other values for the smoothing parameter with coefficient smaller than 0.8 and greater than 1.2 were also considered. In particular the coefficients $3, 2, 1.5, 0.6, 0.5, 0.2$ are considered (and the result are not shown) in this thesis. Unfortunately, the results its –based on the preliminary simulation study– showed that there is no improvements can be obtained when we used for these coefficients. The simulation results when $h_1 = 1.2h$ for the different estimators $\hat{f}_p(0)$ with $p = 1, 1.1, 1.3, 1.5, 2$ and for $\tilde{f}_a^*(0), \tilde{f}_b^*(0), \tilde{f}_c^*(0)$ and $\tilde{f}_d^*(0)$ are given in Tables (2.3) and (2.4), while the results when $h_2 = 0.8h$ are given in Tables (2.5) and (2.6).

Note that $\hat{f}(0) = \hat{f}_1(0)$ in Tables (2.1) and (2.2), while in Tables (2.3) – (2.6), $\hat{f}(0)$ used the optimal rule of h but $\hat{f}_1(0)$ used h multiply by the corresponding coefficient of h.

Question (3) can be answered by inspecting the results in Tables (2.1) – (2.6), where the last four columns in each table give the simulation results related the estimators $\tilde{f}_a^*(0), \tilde{f}_b^*(0), \tilde{f}_c^*(0)$ and $\tilde{f}_d^*(0)$.

To answer Question (5), the estimator $\tilde{f}_m^*(0)$ with $m = 2, 4, 7, 12, 20, 50, 100, 200, 1000,$ and 10000 is studied and the simulation results are presented in Tables (2.7), (2.8) and (2.9).

The simulation results are presented in Tables (2.1) – (2.9).

2.6 SIMULATION RESULTS

By considering the classical kernel estimator $\hat{f}(0)$ as a basis for comparison and based on the simulation results of Tables (2.1) – (2.9), we can conclude the following:

(1) The value of $\hat{f}_p(0)$ increases as p increases $(1 \le p \le 2)$. This is demonstrated in Table (2.1) by inspection the RB of $\hat{f}_p(0)$ when p moves from 1 toward 2 and for each corresponding sample size n. Also, see Tables (2.3) and (2.5).

(2) Despite that the estimator $\hat{f}_2(0)$ performs well in some cases (e.g. EP model with $\beta = 1$ and HR model with $\beta = 1.5$ and 2), it exhibits a poor results –in term of RME– for the other models. The RMEs of $\hat{f}_2(0)$ are –generally– not decreasing when n increases, which indicates that $\hat{f}_2(0)$ is not even a consistent estimator for $f(0)$.

Again and in terms of RME, the performances of $\hat{f}_p(0)$ with $1 < p < 1.5$ seem to be quite well, especially for models that do not have a large shoulder at the origin. In other words, exclude the two models; EP with $\beta = 2.5$ and HR with $\beta = 3.0$, then $\hat{f}_p(0)$ with $1 < p < 1.5$ gives a good results compared to the classical kernel estimator $\hat{f}_1(0)$ (see Table 2.2).

(3) The estimators $\tilde{f}_a^*(0), \tilde{f}_b^*(0), \tilde{f}_c^*(0)$ and $\tilde{f}_d^*(0)$ give very similar performances with overall a slight preference for $\tilde{f}_b^*(0)$. In general, their performances are acceptable comparing to that of $\hat{f}_1(0)$ (see Tables 2.1 and 2.2).

(4) The increasing (or decreasing) in the value of bandwidth (smoothing parameter) from h to $1.2h$ or $0.8h$ improve the performances of the different estimators in some case but in the other cases their performances become worse (see Tables 2.3–2.6). Therefore, we generally recommend to use the rule $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}}$ without any modifications of this rule.

(5) The performances of $\tilde{f}_m^*(0)$ were suprising in most considered cases. Therefore, we selected many values for m to show and to understand how the performances of $\tilde{f}_m^*(0)$ will be for changing value of m. Tables (3.7), (3.8) and (3.9) demonstrate the simulation results of $\tilde{f}_m^*(0)$, at which we introduced the efficiency (EFF) of $\tilde{f}_m^*(0)$ with respect to $\hat{f}_1(0)$ for simple comparison. Note here that $\tilde{f}_m^*(0)$ with $m = 1$ gives the classical kernel estimator $\hat{f}_1(0)$ and the selected values of m were 2, 4, 7, 12, 20, 50, 100, 200, 1000 and 10000. The simulation results demonstrate clearly that $\tilde{f}_m^*(0)$ converges to $\hat{f}_1(0)$ as m becomes 100 or more. In these cases, you can examine the corresponding efficiencies of $\tilde{f}_m^*(0)$ for the three models; EP, HR and BE and for the different values of the sample size n. Despite that, all efficiencies of $\tilde{f}_m^*(0)$ $(m \geq 100)$ are greater than 1, we expect –due to our experience– that these gains in the efficiency are not significant.

(6) For small values of m $(m \leq 4)$, the performance of $\tilde{f}_m^*(0)$ is very acceptable for models that do not have a shoulder at the origin (e.g. EP model with $\beta = 1$ and BE model with different values of β) and also for the models with moderate shoulder at the origin (e.g. EP with $\beta = 1.5$).

(7) The other values of m ($7 \leq m \leq 50$) produced very reasonable estimators. For these cases, the produced estimators seems to be consistent for $f(0)$ in both cases; when the shoulder condition for the simulated model is satisfied and when it is violated. The corresponding efficiencies of $\tilde{f}_m^*(0)$ ($7 \leq m \leq 50$) are considerable in most cases compared to the classical kernel estimator $\hat{f}_1(0)$.

(8) We keep in mind the general good performance of $\tilde{f}_m^*(0)$, this estimator can be implemented in line transect sampling to estimate $f(0)$ and consequently to estimate the abundance D according to the following rule:

If the collected perpendicular distances $x_1, x_2, \ldots x_n$ seem to be spike at the origin (i.e. the shoulder condition is not satisfied) then $\tilde{f}_m^*(0)$ with m around the value 4 can be used. If the perpendicular distances seem to be have a shoulder condition at the origin then $\tilde{f}_m^*(0)$ with a value of m around 15 can be used. If there is no information about the model shape of the perpendicular distances then we recommend to use $\tilde{f}_m^*(0)$ with a value of m around 10.

Finally, it is worthwhile to mention here that the shoulder condition of the data can be checked by using the informal histogram method with 4 to 10 intervals (see Buckland et. al. 2001) or by using the formal method of Zhang (2001).

2.7 DISCUSSION

We were planning in the first stage (phase) of this thesis to study the estimator $\hat{f}_p(0)$ and to investigate the effect of varying p and h on its performances.

The first question that was in our minds; can we improve the performance of the classical kernel estimator $\hat{f}(0)$ (equation 1.5) by considering $\hat{f}_p(0)$ and then varying the value of p ?

The second question was; when we decide the best value(s) of p, can we improve the corresponding estimator by varying the value of the bandwidth (smoothing parameter) h ?

The second question came to our mind due to the well known result that says: h plays an important role in the performance of the kernel estimator (see Silverman, 1986 and Gerard and Schucany, 1999). After we preparing our simulation programs by using the same detection functions that stated in Section (2.5), we run our programs by considering the estimators $\hat{f}(0)$ and $\hat{f}_p(0)$ with $p = -2, -1, 0, 1, 1.5, 2, 3, 4$ and then for each value of p we used the rule $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}}$ which recommended by Gerard and Schucany (1999) to compute the smoothing parameter. The results showed that $\hat{f}_p(0)$ gives very bad results when $p < 1$ as well as when $p > 2$ (many other non-integer values for p are also tried). After that we tried to improve the performance of $\hat{f}_p(0)$ by varying the value of h. We were used the new values of the smoothing parameter, *3h, 2h, 1.5h, 1.2h, 0.8h, 0.6h, 0.5h* and *0.2h*. Unfortunately, the results of the preliminary simulation study showed that the performances of $\hat{f}_p(0)$ cannot be improved (by varying h) for most values of p, specially when $p < 0.5$ and when $p > 3$. However, we found that the case that deserves to pursue is the estimator $\hat{f}_p(0)$ when $1 \le p \le 2$. Therefore, most results of the preliminary simulation study are not presented in this work.

One may concludes that the preliminary simulation study was useless. But we believe that it was very important for many reasons. Firstly, it tells the researchers to avoid studying the other measure of centers for $Y_1, Y_2, \dots, Y_n$ (see Section 2.2) and the mean of $Y_1, Y_2, \dots, Y_n$ (classical kernel estimator of $f(0)$) remains very reasonable and acceptable compared to other measures. Secondly, some important results of the preliminary study are given in Section (2.3). Finally, the preliminary study together with Theorem (2.1) lead us to the idea of forming estimator (2.3). This estimator –as the simulation results shown– is very important and it gives a very worthwhile results compared to the classical kernel estimator.

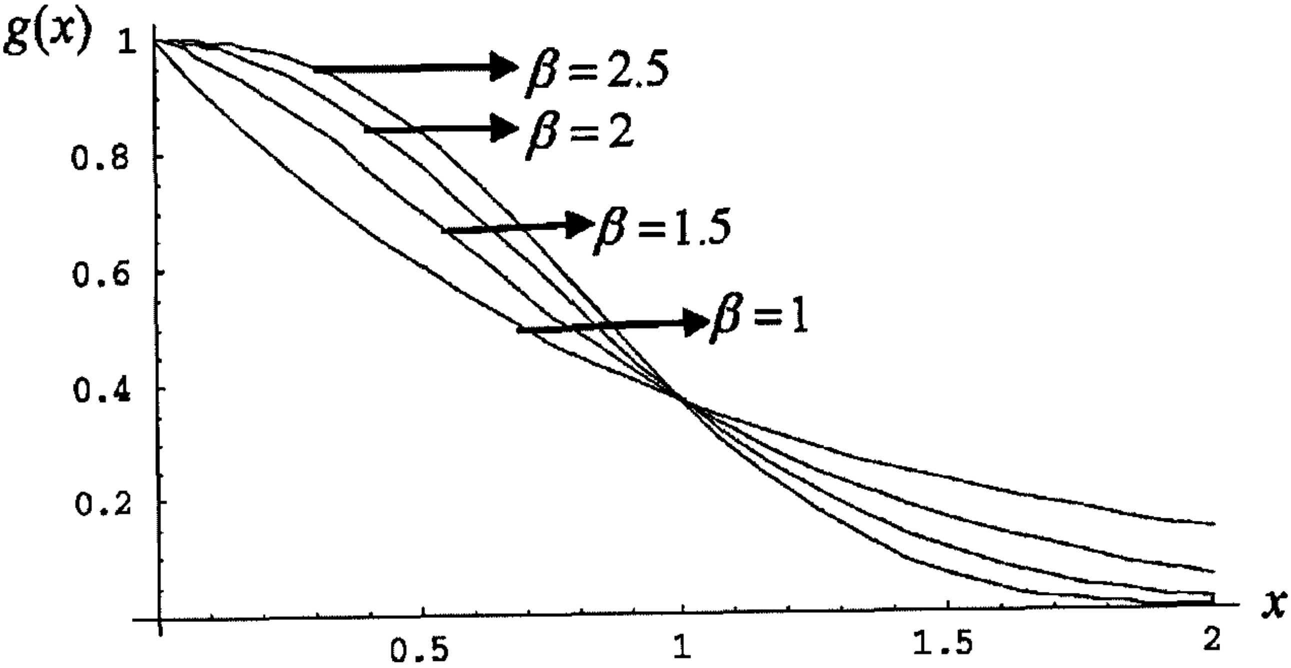


Figure (2.1): Exponential power detection function, $g(x)$ for β=1.0, 1.5, 2.0, and 2.5.

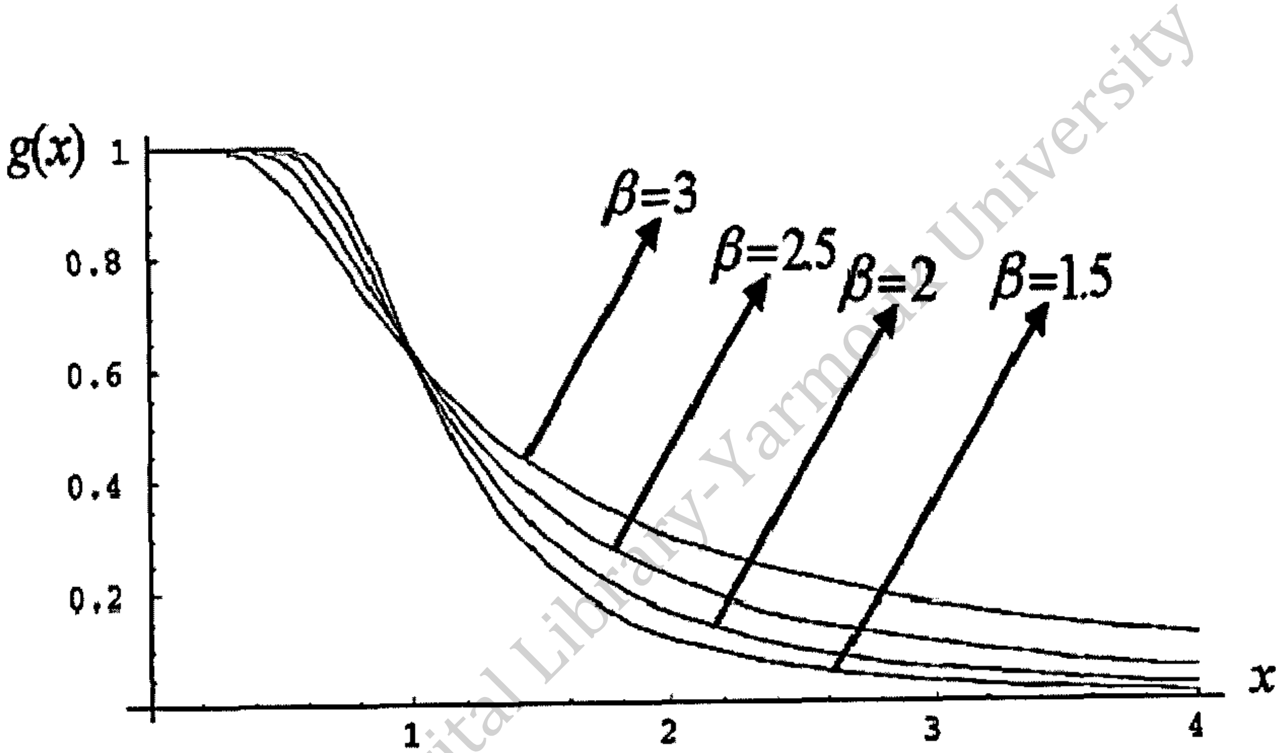


Figure (2.2): Hazard-rate detection function, $g(x)$ for β=1.5, 2.0, 2.5, *and* 3.0.

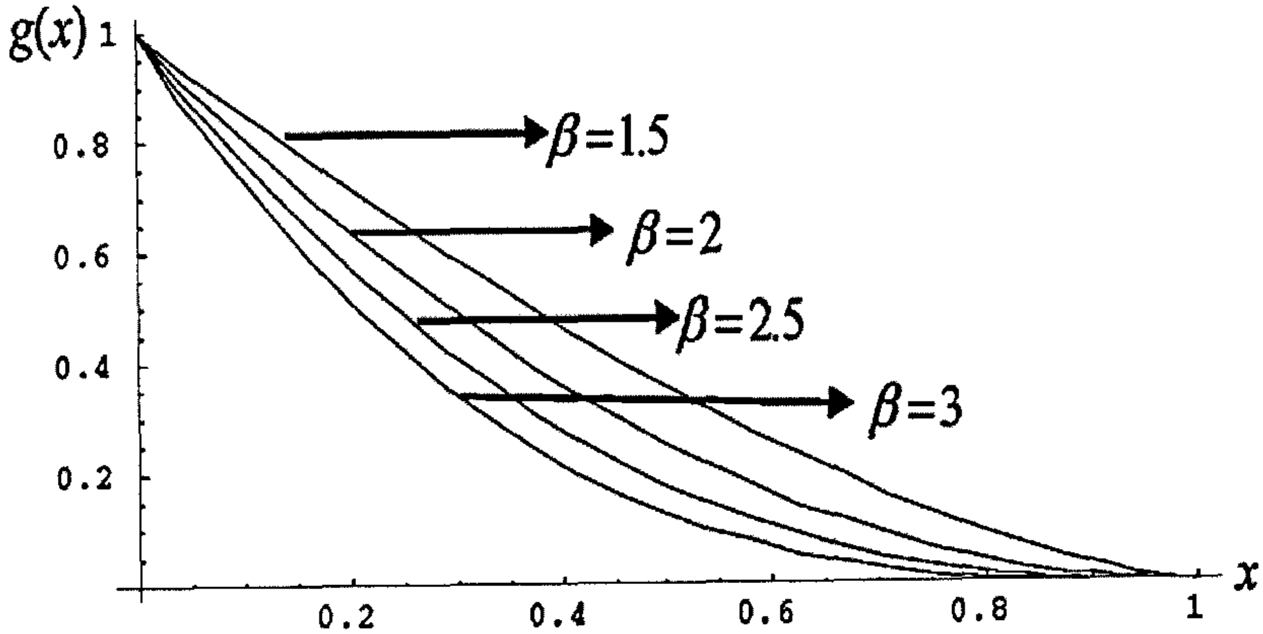


Figure (2.3): Beta detection function, $g(x)$ for β=1.5, 2.0, 2.5, *and* 3.0.

Table (2.1): The simulated value of the relative bias (RB) for each estimator by using the bandwidth $h = 1.06\,\hat{\sigma}\, n^{-\frac{1}{5}}$

Exponential power Model	n	$\hat{f}_1(0)$	$\hat{f}_{1.1}(0)$	$\hat{f}_{1.3}(0)$	$\hat{f}_{1.5}(0)$	$\hat{f}_2(0)$	$\tilde{f}_a^*(0)$	$\tilde{f}_b^*(0)$	$\tilde{f}_c^*(0)$	$\tilde{f}_d^*(0)$
$\beta = 1$	50	-0.35	-0.33	-0.29	-0.26	-0.19	-0.29	-0.29	-0.31	-0.32
$w = 5$	100	-0.32	-0.29	-0.24	-0.20	-0.12	-0.24	-0.25	-0.27	-0.28
	200	-0.29	-0.25	-0.20	-0.15	-0.05	-0.19	-0.20	-0.23	-0.25
$\beta = 1.5$	50	-0.18	-0.15	-0.09	-0.04	0.05	-0.09	-0.09	-0.12	-0.13
$w = 3$	100	-0.15	-0.11	-0.04	0.02	0.14	-0.03	-0.04	-0.08	-0.10
	200	-0.13	-0.08	0.01	0.08	0.23	0.02	0.00	-0.05	-0.07
$\beta = 2$	50	-0.09	-0.06	0.01	0.07	0.19	0.02	0.01	-0.02	-0.04
$w = 2.5$	100	-0.08	-0.03	0.05	0.13	0.27	0.06	0.05	0.01	-0.02
	200	-0.06	0.00	0.10	0.19	0.37	0.11	0.09	0.03	0.01
$\beta = 2.5$	50	-0.05	-0.01	0.06	0.12	0.26	0.07	0.06	0.03	0.01
$w = 2$	100	-0.04	0.01	0.10	0.18	0.35	0.11	0.10	0.05	0.02
	200	-0.03	0.03	0.15	0.24	0.45	0.16	0.14	0.07	0.04
Hazard Rate Model										
$\beta = 1.5$	50	-0.42	-0.40	-0.38	-0.36	-0.32	-0.38	-0.38	-0.39	-0.40
$w = 20$	100	-0.38	-0.36	-0.33	-0.31	-0.27	-0.33	-0.34	-0.35	-0.36
	200	-0.33	-0.31	-0.27	-0.24	-0.19	-0.27	-0.28	-0.29	-0.30
$\beta = 2$	50	-0.26	-0.24	-0.21	-0.19	-0.14	-0.21	-0.21	-0.22	-0.23
$w = 12$	100	-0.22	-0.20	-0.16	-0.13	-0.07	-0.16	-0.17	-0.18	-0.19
	200	-0.18	-0.15	-0.11	-0.07	0.00	-0.10	-0.11	-0.13	-0.14
$\beta = 2.5$	50	-0.13	-0.10	-0.06	-0.03	0.05	-0.06	-0.06	-0.07	-0.08
$w = 8$	100	-0.10	-0.07	-0.02	0.03	0.12	-0.01	-0.02	-0.03	-0.05
	200	-0.06	-0.03	0.04	0.09	0.20	0.04	0.03	0.01	-0.01
$\beta = 3$	50	-0.06	-0.04	0.02	0.06	0.15	0.02	0.02	0.01	-0.01
$w = 6$	100	-0.04	0.00	0.06	0.12	0.24	0.07	0.07	0.04	0.02
	200	-0.02	0.03	0.11	0.18	0.32	0.12	0.11	0.07	0.05
Beta Model										
$\beta = 1.5$	50	-0.19	-0.15	-0.09	-0.04	0.07	-0.09	-0.09	-0.12	-0.14
$w = 1$	100	-0.17	-0.13	-0.05	0.02	0.15	-0.04	-0.05	-0.10	-0.12
	200	-0.15	-0.10	-0.01	0.08	0.24	0.01	-0.01	-0.07	-0.09
$\beta = 2$	50	-0.21	-0.18	-0.12	-0.07	0.03	-0.12	-0.13	-0.15	-0.17
$w = 1$	100	-0.19	-0.15	-0.08	-0.02	0.10	-0.07	-0.09	-0.12	-0.14
	200	-0.17	-0.12	-0.03	0.04	0.19	-0.02	-0.04	-0.09	-0.11
$\beta = 2.5$	50	-0.23	-0.20	-0.15	-0.10	-0.01	-0.14	-0.15	-0.17	-0.19
$w = 1$	100	-0.21	-0.18	-0.11	-0.05	0.06	-0.10	-0.11	-0.15	-0.17
	200	-0.19	-0.14	-0.06	0.02	0.16	-0.05	-0.06	-0.11	-0.13
$\beta = 3$	50	-0.25	-0.22	-0.17	-0.13	-0.04	-0.17	-0.17	-0.20	-0.21
$w = 1$	100	-0.22	-0.19	-0.12	-0.07	0.04	-0.12	-0.13	-0.16	-0.18
	200	-0.20	-0.15	-0.08	-0.01	0.13	-0.07	-0.08	-0.13	-0.15

Table (2.2): The simulated value of the relative mean error (RME) for each estimator by using the bandwidth $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}}$

Exponential power Model	n	$\hat{f}_1(0)$	$\hat{f}_{1.1}(0)$	$\hat{f}_{1.3}(0)$	$\hat{f}_{1.5}(0)$	$\hat{f}_2(0)$	$\tilde{f}_a^*(0)$	$\tilde{f}_b^*(0)$	$\tilde{f}_c^*(0)$	$\tilde{f}_d^*(0)$
$\beta = 1$	50	0.36	0.34	0.31	0.28	0.22	0.30	0.31	0.32	0.33
$w = 5$	100	0.33	0.30	0.25	0.22	0.15	0.25	0.26	0.28	0.29
	200	0.29	0.26	0.20	0.16	0.08	0.20	0.21	0.24	0.25
$\beta = 1.5$	50	0.21	0.19	0.15	0.13	0.14	0.15	0.15	0.16	0.18
$w = 3$	100	0.17	0.14	0.10	0.10	0.18	0.10	0.10	0.12	0.13
	200	0.14	0.10	0.07	0.11	0.24	0.08	0.07	0.08	0.10
$\beta = 2$	50	0.15	0.14	0.13	0.15	0.24	0.13	0.13	0.13	0.13
$w = 2.5$	100	0.12	0.10	0.11	0.16	0.29	0.12	0.11	0.10	0.10
	200	0.09	0.08	0.13	0.21	0.38	0.14	0.12	0.08	0.08
$\beta = 2.5$	50	0.14	0.13	0.15	0.19	0.30	0.16	0.15	0.13	0.13
$w = 2$	100	0.11	0.10	0.15	0.21	0.37	0.15	0.14	0.11	0.10
	200	0.08	0.09	0.17	0.26	0.46	0.14	0.13	0.11	0.09
Hazard Rate Model										
$\beta = 1.5$	50	0.43	0.42	0.40	0.38	0.35	0.39	0.40	0.41	0.41
$w = 20$	100	0.39	0.37	0.34	0.32	0.28	0.34	0.35	0.36	0.37
	200	0.33	0.31	0.28	0.25	0.20	0.28	0.28	0.30	0.31
$\beta = 2$	50	0.29	0.28	0.25	0.23	0.21	0.25	0.25	0.26	0.27
$w = 12$	100	0.24	0.22	0.19	0.16	0.13	0.19	0.19	0.20	0.21
	200	0.19	0.16	0.13	0.10	0.08	0.12	0.13	0.14	0.15
$\beta = 2.5$	50	0.18	0.16	0.15	0.14	0.16	0.15	0.15	0.15	0.15
$w = 8$	100	0.13	0.11	0.10	0.11	0.16	0.10	0.10	0.10	0.10
	200	0.09	0.07	0.08	0.12	0.22	0.08	0.08	0.07	0.07
$\beta = 3$	50	0.14	0.13	0.14	0.16	0.22	0.14	0.14	0.13	0.13
$w = 6$	100	0.10	0.09	0.12	0.16	0.27	0.12	0.13	0.10	0.10
	200	0.07	0.07	0.13	0.20	0.34	0.12	0.13	0.10	0.08
Beta Model										
$\beta = 1.5$	50	0.22	0.19	0.15	0.13	0.15	0.15	0.15	0.17	0.18
$w = 1$	100	0.19	0.15	0.10	0.09	0.18	0.10	0.10	0.13	0.14
	200	0.17	0.12	0.07	0.11	0.25	0.07	0.07	0.10	0.12
$\beta = 2$	50	0.24	0.21	0.17	0.14	0.13	0.17	0.17	0.19	0.20
$w = 1$	100	0.21	0.18	0.12	0.10	0.14	0.12	0.12	0.15	0.17
	200	0.18	0.14	0.08	0.09	0.21	0.08	0.08	0.12	0.13
$\beta = 2.5$	50	0.26	0.23	0.19	0.16	0.13	0.19	0.19	0.21	0.22
$w = 1$	100	0.23	0.20	0.14	0.10	0.11	0.13	0.14	0.17	0.19
	200	0.20	0.15	0.09	0.07	0.18	0.08	0.09	0.13	0.15
$\beta = 3$	50	0.27	0.25	0.20	0.17	0.13	0.20	0.20	0.22	0.24
$w = 1$	100	0.24	0.21	0.15	0.11	0.11	0.15	0.15	0.18	0.20
	200	0.21	0.17	0.10	0.07	0.15	0.09	0.10	0.14	0.16

Table (2.3): The simulated value of the relative bias (RB) for each estimator by using the bandwidth $h_1 = 1.2h$

Exponential power Model	n	$\hat{f}(0)$	$\hat{f}_1(0)$	$\hat{f}_{1.1}(0)$	$\hat{f}_{1.3}(0)$	$\hat{f}_{1.5}(0)$	$\hat{f}_2(0)$	$\tilde{f}_a^*(0)$	$\tilde{f}_b^*(0)$	$\tilde{f}_c^*(0)$	$\tilde{f}_d^*(0)$
$\beta = 1$	50	-0.35	-0.40	-0.38	-0.35	-0.33	-0.28	-0.35	-0.35	-0.36	-0.37
$w = 5$	100	-0.32	-0.36	-0.34	-0.31	-0.28	-0.22	-0.31	-0.31	-0.32	-0.33
	200	-0.29	-0.33	-0.31	-0.26	-0.22	-0.15	-0.26	-0.26	-0.28	-0.30
$\beta = 1.5$	50	-0.18	-0.22	-0.20	-0.16	-0.12	-0.06	-0.16	-0.16	-0.17	-0.18
$w = 3$	100	-0.15	-0.19	-0.16	-0.11	-0.06	0.03	-0.10	-0.11	-0.13	-0.14
	200	-0.13	-0.16	-0.12	-0.06	0.00	0.11	-0.05	-0.06	-0.09	-0.11
$\beta = 2$	50	-0.09	-0.13	-0.10	-0.05	-0.01	0.07	-0.05	-0.05	-0.06	-0.08
$w = 2.5$	100	-0.08	-0.11	-0.07	-0.01	0.04	0.15	0.00	-0.01	-0.03	-0.05
	200	-0.06	-0.08	-0.04	0.04	0.10	0.24	0.05	0.04	0.00	-0.02
$\beta = 2.5$	50	-0.05	-0.08	-0.05	0.00	0.05	0.14	0.00	0.00	-0.01	-0.03
$w = 2$	100	-0.04	-0.06	-0.03	0.04	0.10	0.22	0.05	0.04	0.02	-0.01
	200	-0.03	-0.05	0.00	0.09	0.16	0.32	0.10	0.09	0.04	0.02
Hazard Rate Model											
$\beta = 1.5$	50	-0.42	-0.48	-0.47	-0.45	-0.44	-0.41	-0.45	-0.46	-0.46	-0.47
$w = 20$	100	-0.38	-0.45	-0.43	-0.41	-0.39	-0.36	-0.41	-0.41	-0.42	-0.43
	200	-0.33	-0.40	-0.38	-0.35	-0.33	-0.29	-0.35	-0.36	-0.37	-0.38
$\beta = 2$	50	-0.26	-0.33	-0.32	-0.29	-0.28	-0.24	-0.29	-0.30	-0.30	-0.30
$w = 12$	100	-0.22	-0.29	-0.27	-0.25	-0.22	-0.18	-0.24	-0.25	-0.25	-0.26
	200	-0.18	-0.24	-0.22	-0.19	-0.16	-0.11	-0.19	-0.19	-0.20	-0.21
$\beta = 2.5$	50	-0.13	-0.19	-0.17	-0.14	-0.12	-0.07	-0.14	-0.14	-0.14	-0.15
$w = 8$	100	-0.10	-0.15	-0.13	-0.09	-0.06	0.00	-0.09	-0.09	-0.10	-0.11
	200	-0.06	-0.11	-0.08	-0.04	0.00	0.08	-0.03	-0.04	-0.05	-0.06
$\beta = 3$	50	-0.06	-0.11	-0.09	-0.05	-0.02	0.04	-0.05	-0.05	-0.05	-0.06
$w = 6$	100	-0.04	-0.08	-0.05	0.00	0.04	0.12	0.00	0.00	-0.01	-0.02
	200	-0.02	-0.05	-0.01	0.05	0.10	0.20	0.05	0.05	0.03	0.01
Beta Model											
$\beta = 1.5$	50	-0.19	-0.22	-0.20	-0.16	-0.12	-0.04	-0.15	-0.15	-0.17	-0.18
$w = 1$	100	-0.17	-0.20	-0.17	-0.11	-0.06	0.04	-0.11	-0.11	-0.14	-0.15
	200	-0.15	-0.18	-0.14	-0.07	-0.01	0.12	-0.06	-0.07	-0.11	-0.13
$\beta = 2$	50	-0.21	-0.25	-0.23	-0.19	-0.15	-0.08	-0.18	-0.19	-0.20	-0.21
$w = 1$	100	-0.19	-0.23	-0.20	-0.15	-0.10	-0.01	-0.14	-0.15	-0.17	-0.18
	200	-0.17	-0.20	-0.16	-0.10	-0.04	0.08	-0.09	-0.10	-0.13	-0.15
$\beta = 2.5$	50	-0.23	-0.27	-0.25	-0.21	-0.18	-0.11	-0.21	-0.21	-0.22	-0.24
$w = 1$	100	-0.21	-0.25	-0.22	-0.17	-0.13	-0.04	-0.17	-0.17	-0.20	-0.21
	200	-0.19	-0.22	-0.18	-0.12	-0.07	0.04	-0.11	-0.12	-0.15	-0.17
$\beta = 3$	50	-0.25	-0.29	-0.27	-0.23	-0.20	-0.14	-0.23	-0.23	-0.25	-0.26
$w = 1$	100	-0.22	-0.26	-0.23	-0.19	-0.14	-0.06	-0.18	-0.19	-0.21	-0.22
	200	-0.20	-0.23	-0.20	-0.14	-0.09	0.02	-0.13	-0.14	-0.17	-0.19

Table (2.4): The simulated value of the relative mean error (RME) for each estimator by using the bandwidth $h_1 = 1.2h$

Exponential power Model	n	$f(0)$	$\hat{f}_1(0)$	$\hat{f}_{1.1}(0)$	$\hat{f}_{1.3}(0)$	$\hat{f}_{1.5}(0)$	$\hat{f}_2(0)$	$\bar{f}^*_a(0)$	$\bar{f}^*_b(0)$	$\bar{f}^*_c(0)$	$\bar{f}^*_d(0)$
$\beta = 1$, $w = 5$	50	0.36	0.40	0.39	0.36	0.34	0.30	0.36	0.36	0.37	0.38
	100	0.33	0.37	0.35	0.31	0.29	0.23	0.31	0.32	0.33	0.34
	200	0.29	0.33	0.31	0.27	0.23	0.16	0.26	0.27	0.29	0.30
$\beta = 1.5$, $w = 3$	50	0.21	0.24	0.22	0.19	0.16	0.13	0.19	0.19	0.20	0.21
	100	0.17	0.20	0.18	0.13	0.11	0.10	0.13	0.13	0.15	0.16
	200	0.14	0.17	0.14	0.09	0.07	0.13	0.08	0.09	0.11	0.12
$\beta = 2$, $w = 2.5$	50	0.15	0.17	0.15	0.13	0.12	0.14	0.13	0.12	0.13	0.14
	100	0.12	0.13	0.11	0.09	0.10	0.18	0.09	0.08	0.09	0.10
	200	0.09	0.10	0.08	0.08	0.13	0.25	0.08	0.08	0.07	0.07
$\beta = 2.5$, $w = 2$	50	0.14	0.14	0.12	0.12	0.13	0.19	0.12	0.11	0.11	0.12
	100	0.11	0.11	0.09	0.10	0.14	0.24	0.10	0.10	0.09	0.09
	200	0.08	0.08	0.07	0.11	0.18	0.33	0.10	0.09	0.08	0.07
Hazard Rate Model											
$\beta = 1.5$, $w = 20$	50	0.43	0.49	0.48	0.47	0.45	0.43	0.46	0.47	0.47	0.48
	100	0.39	0.45	0.44	0.42	0.40	0.37	0.42	0.42	0.43	0.43
	200	0.33	0.40	0.38	0.36	0.34	0.30	0.36	0.36	0.37	0.38
$\beta = 2$, $w = 12$	50	0.29	0.35	0.34	0.32	0.31	0.28	0.32	0.32	0.33	0.33
	100	0.24	0.30	0.29	0.26	0.24	0.20	0.26	0.26	0.27	0.27
	200	0.19	0.25	0.23	0.20	0.18	0.13	0.20	0.20	0.21	0.22
$\beta = 2.5$, $w = 8$	50	0.18	0.22	0.21	0.19	0.18	0.16	0.19	0.19	0.19	0.20
	100	0.13	0.17	0.15	0.13	0.11	0.10	0.13	0.13	0.13	0.14
	200	0.09	0.13	0.11	0.08	0.07	0.11	0.08	0.08	0.08	0.09
$\beta = 3$, $w = 6$	50	0.14	0.16	0.15	0.14	0.13	0.15	0.13	0.13	0.13	0.14
	100	0.10	0.11	0.10	0.09	0.10	0.16	0.09	0.09	0.09	0.09
	200	0.07	0.07	0.06	0.08	0.12	0.22	0.08	0.08	0.07	0.06
Beta Model											
$\beta = 1.5$, $w = 1$	50	0.22	0.25	0.22	0.19	0.16	0.12	0.19	0.19	0.20	0.21
	100	0.19	0.21	0.18	0.14	0.10	0.09	0.13	0.13	0.16	0.17
	200	0.17	0.19	0.15	0.09	0.07	0.14	0.09	0.09	0.12	0.14
$\beta = 2$, $w = 1$	50	0.24	0.27	0.25	0.21	0.18	0.13	0.21	0.21	0.22	0.23
	100	0.21	0.24	0.21	0.17	0.13	0.08	0.16	0.16	0.19	0.20
	200	0.18	0.21	0.17	0.12	0.08	0.10	0.11	0.12	0.15	0.16
$\beta = 2.5$, $w = 1$	50	0.26	0.29	0.27	0.24	0.21	0.16	0.23	0.23	0.25	0.26
	100	0.23	0.26	0.23	0.19	0.15	0.09	0.19	0.19	0.21	0.22
	200	0.20	0.23	0.19	0.13	0.09	0.08	0.13	0.14	0.17	0.18
$\beta = 3$, $w = 1$	50	0.27	0.31	0.29	0.25	0.23	0.17	0.25	0.25	0.26	0.28
	100	0.24	0.27	0.25	0.20	0.17	0.10	0.20	0.20	0.22	0.23
	200	0.21	0.24	0.21	0.15	0.11	0.07	0.15	0.15	0.18	0.20

Table (2.5): The simulated value of the relative bias (RB) for each estimator by using the bandwidth $h_2 = 0.8h$

Exponential power Model	n	$\hat{f}(0)$	$\hat{f}_1(0)$	$\hat{f}_{1.1}(0)$	$\hat{f}_{1.3}(0)$	$\hat{f}_{1.5}(0)$	$\hat{f}_2(0)$	$\tilde{f}_a^*(0)$	$\tilde{f}_b^*(0)$	$\tilde{f}_c^*(0)$	$\tilde{f}_d^*(0)$
$\beta = 1$, $w = 5$	50	-0.35	-0.30	-0.27	-0.21	-0.16	-0.07	-0.20	-0.21	-0.24	-0.26
	100	-0.32	-0.27	-0.23	-0.16	-0.11	0.01	-0.16	-0.17	-0.21	-0.23
	200	-0.29	-0.24	-0.20	-0.12	-0.05	0.09	-0.10	-0.12	-0.17	-0.19
$\beta = 1.5$, $w = 3$	50	-0.18	-0.14	-0.09	-0.02	0.05	0.19	-0.01	-0.02	-0.06	-0.08
	100	-0.15	-0.11	-0.06	0.04	0.12	0.29	0.05	0.03	-0.03	-0.05
	200	-0.13	-0.09	-0.03	0.09	0.19	0.39	0.11	0.08	0.00	-0.03
$\beta = 2$, $w = 2.5$	50	-0.09	-0.06	-0.01	0.08	0.17	0.34	0.10	0.08	0.03	0.00
	100	-0.08	-0.05	0.01	0.13	0.23	0.43	0.14	0.12	0.05	0.02
	200	-0.06	-0.04	0.04	0.17	0.29	0.54	0.20	0.16	0.07	0.04
$\beta = 2.5$, $w = 2$	50	-0.05	-0.03	0.03	0.13	0.22	0.41	0.14	0.13	0.07	0.04
	100	-0.04	-0.03	0.04	0.17	0.28	0.51	0.19	0.16	0.08	0.05
	200	-0.03	-0.02	0.06	0.22	0.35	0.62	0.25	0.21	0.10	0.06
Hazard Rate Model											
$\beta = 1.5$, $w = 20$	50	-0.42	-0.33	-0.31	-0.28	-0.25	-0.20	-0.28	-0.28	-0.30	-0.31
	100	-0.38	-0.29	-0.27	-0.23	-0.20	-0.13	-0.23	-0.24	-0.26	-0.26
	200	-0.33	-0.24	-0.21	-0.16	-0.12	-0.05	-0.16	-0.17	-0.20	-0.21
$\beta = 2$, $w = 12$	50	-0.26	-0.18	-0.15	-0.11	-0.07	0.00	-0.11	-0.11	-0.13	-0.14
	100	-0.22	-0.14	-0.11	-0.06	-0.02	0.07	-0.06	-0.07	-0.08	-0.10
	200	-0.18	-0.11	-0.07	-0.01	0.05	0.15	0.00	-0.01	-0.04	-0.06
$\beta = 2.5$, $w = 8$	50	-0.13	-0.07	-0.03	0.03	0.08	0.19	0.03	0.03	0.01	-0.01
	100	-0.10	-0.05	0.00	0.07	0.13	0.26	0.08	0.07	0.04	0.02
	200	-0.06	-0.02	0.03	0.12	0.19	0.35	0.13	0.11	0.07	0.04
$\beta = 3$, $w = 6$	50	-0.06	-0.03	0.02	0.09	0.16	0.30	0.10	0.10	0.06	0.04
	100	-0.04	-0.01	0.04	0.13	0.22	0.39	0.15	0.13	0.09	0.06
	200	-0.02	0.00	0.06	0.18	0.28	0.48	0.19	0.17	0.11	0.07
Beta Model											
$\beta = 1.5$, $w = 1$	50	-0.19	-0.15	-0.10	-0.02	0.06	0.21	-0.01	-0.02	-0.07	-0.10
	100	-0.17	-0.14	-0.08	0.02	0.11	0.30	0.04	0.02	-0.05	-0.08
	200	-0.15	-0.12	-0.05	0.07	0.18	0.40	0.09	0.06	-0.03	-0.06
$\beta = 2$, $w = 1$	50	-0.21	-0.18	-0.13	-0.05	0.02	0.17	-0.04	-0.05	-0.10	-0.12
	100	-0.19	-0.16	-0.10	-0.01	0.08	0.25	0.01	-0.01	-0.08	-0.10
	200	-0.17	-0.14	-0.07	0.04	0.15	0.35	0.07	0.03	-0.05	-0.07
$\beta = 2.5$, $w = 1$	50	-0.23	-0.19	-0.15	-0.07	-0.01	0.13	-0.06	-0.08	-0.12	-0.14
	100	-0.21	-0.18	-0.13	-0.03	0.05	0.21	-0.02	-0.04	-0.10	-0.12
	200	-0.19	-0.15	-0.09	0.02	0.12	0.31	0.04	0.01	-0.06	-0.09
$\beta = 3$, $w = 1$	50	-0.25	-0.21	-0.17	-0.09	-0.03	0.10	-0.08	-0.10	-0.14	-0.16
	100	-0.22	-0.18	-0.13	-0.05	0.03	0.19	-0.03	-0.05	-0.11	-0.13
	200	-0.20	-0.16	-0.10	0.00	0.10	0.28	0.02	-0.01	-0.08	-0.10

Table (2.6): The simulated value of the relative mean error (RME) for each estimator by using the bandwidth $h_2 = 0.8h$

Exponential power Model	n	$f(0)$	$\hat{f}_1(0)$	$\hat{f}_{1.1}(0)$	$\hat{f}_{1.3}(0)$	$\hat{f}_{1.5}(0)$	$\hat{f}_2(0)$	$\tilde{f}_a^*(0)$	$\tilde{f}_b^*(0)$	$\tilde{f}_c^*(0)$	$\tilde{f}_d^*(0)$
$\beta = 1$, $w = 5$	50	0.36	0.32	0.29	0.24	0.20	0.14	0.23	0.24	0.27	0.28
	100	0.33	0.28	0.25	0.19	0.14	0.10	0.18	0.19	0.23	0.24
	200	0.29	0.25	0.21	0.13	0.08	0.12	0.12	0.14	0.19	0.20
$\beta = 1.5$, $w = 3$	50	0.21	0.19	0.16	0.14	0.16	0.25	0.14	0.14	0.15	0.16
	100	0.17	0.15	0.12	0.12	0.17	0.32	0.13	0.12	0.11	0.12
	200	0.14	0.12	0.09	0.12	0.21	0.40	0.14	0.11	0.08	0.09
$\beta = 2$, $w = 2.5$	50	0.15	0.16	0.15	0.18	0.23	0.38	0.19	0.18	0.15	0.15
	100	0.12	0.12	0.11	0.17	0.26	0.45	0.19	0.17	0.12	0.12
	200	0.09	0.10	0.10	0.20	0.31	0.55	0.22	0.19	0.11	0.10
$\beta = 2.5$, $w = 2$	50	0.14	0.16	0.16	0.21	0.28	0.45	0.22	0.21	0.17	0.16
	100	0.11	0.12	0.13	0.21	0.31	0.53	0.23	0.21	0.15	0.13
	200	0.08	0.09	0.12	0.24	0.37	0.64	0.27	0.23	0.14	0.11
Hazard Rate Model											
$\beta = 1.5$, $w = 20$	50	0.43	0.35	0.34	0.31	0.28	0.24	0.30	0.31	0.33	0.33
	100	0.39	0.31	0.28	0.25	0.22	0.17	0.24	0.25	0.27	0.28
	200	0.33	0.25	0.22	0.18	0.14	0.09	0.17	0.18	0.21	0.22
$\beta = 2$, $w = 12$	50	0.29	0.23	0.21	0.19	0.17	0.17	0.18	0.19	0.19	0.20
	100	0.24	0.17	0.15	0.12	0.11	0.14	0.11	0.12	0.13	0.14
	200	0.19	0.13	0.10	0.07	0.09	0.17	0.07	0.07	0.08	0.09
$\beta = 2.5$, $w = 8$	50	0.18	0.15	0.14	0.15	0.18	0.26	0.15	0.15	0.14	0.14
	100	0.13	0.11	0.10	0.13	0.18	0.29	0.13	0.13	0.11	0.10
	200	0.09	0.08	0.08	0.14	0.21	0.36	0.15	0.14	0.10	0.09
$\beta = 3$, $w = 6$	50	0.14	0.14	0.15	0.18	0.23	0.35	0.19	0.18	0.16	0.15
	100	0.10	0.10	0.11	0.18	0.25	0.41	0.19	0.18	0.14	0.12
	200	0.07	0.08	0.10	0.20	0.29	0.49	0.21	0.19	0.13	0.11
Beta Model											
$\beta = 1.5$, $w = 1$	50	0.22	0.20	0.17	0.14	0.16	0.26	0.14	0.14	0.16	0.17
	100	0.19	0.17	0.13	0.11	0.16	0.32	0.12	0.11	0.11	0.13
	200	0.17	0.15	0.10	0.11	0.20	0.41	0.13	0.10	0.09	0.10
$\beta = 2$, $w = 1$	50	0.24	0.22	0.19	0.15	0.15	0.23	0.15	0.15	0.17	0.18
	100	0.21	0.19	0.15	0.11	0.14	0.28	0.11	0.11	0.13	0.15
	200	0.18	0.16	0.11	0.10	0.17	0.37	0.11	0.09	0.10	0.11
$\beta = 2.5$, $w = 1$	50	0.26	0.23	0.20	0.16	0.14	0.20	0.15	0.16	0.18	0.19
	100	0.23	0.20	0.16	0.11	0.12	0.24	0.11	0.11	0.14	0.16
	200	0.20	0.17	0.12	0.09	0.15	0.33	0.09	0.08	0.10	0.12
$\beta = 3$, $w = 1$	50	0.27	0.24	0.21	0.16	0.14	0.18	0.16	0.16	0.19	0.20
	100	0.24	0.21	0.17	0.11	0.11	0.22	0.11	0.12	0.15	0.16
	200	0.21	0.18	0.13	0.08	0.13	0.30	0.08	0.08	0.11	0.13

Table (2.7): The relative bias (RB), relative mean error (RME) and efficiency (EFF) of the proposed estimator $\tilde{f}^*_m(0)$ with m = 2, 4, 7, 12, 20, 50, 100, 200, 1000 and 10000 when the data are simulated from Exponential Power (EP) Model.

Exponential power Model	m	1	2	4	7	12	20	50	100	200	1000	10000
n = 50	RB	-0.354	-0.282	-0.289	-0.304	-0.318	-0.329	-0.341	-0.346	-0.350	-0.353	-0.354
β = 1.0	RME	0.366	0.299	0.305	0.320	0.333	0.342	0.354	0.359	0.362	0.365	0.366
w = 5.0	EFF		**1.224**	**1.199**	**1.145**	**1.100**	**1.068**	**1.034**	**1.019**	**1.011**	**1.003**	**1.000**
n = 100	RB	-0.319	-0.231	-0.242	-0.261	-0.278	-0.291	-0.305	-0.311	-0.314	-0.318	-0.319
β = 1.0	RME	0.327	0.244	0.253	0.272	0.288	0.300	0.313	0.319	0.322	0.326	0.327
w = 5.0	EFF		**1.341**	**1.290**	**1.201**	**1.134**	**1.090**	**1.043**	**1.025**	**1.014**	**1.003**	**1.000**
n = 200	RB	-0.288	-0.183	-0.198	-0.222	-0.242	-0.256	-0.272	-0.279	-0.283	-0.287	-0.288
β = 1.0	RME	0.293	0.193	0.206	0.230	0.249	0.263	0.278	0.285	0.288	0.292	0.293
w = 5.0	EFF		**1.522**	**1.420**	**1.277**	**1.179**	**1.117**	**1.055**	**1.031**	**1.017**	**1.004**	**1.001**
n = 50	RB	-0.172	-0.064	-0.073	-0.097	-0.119	-0.134	-0.153	-0.161	-0.166	-0.170	-0.171
β = 1.5	RME	0.203	0.132	0.136	0.150	0.164	0.175	0.189	0.195	0.199	0.202	0.203
w = 3.0	EFF		**1.541**	**1.496**	**1.360**	**1.242**	**1.162**	**1.077**	**1.043**	**1.024**	**1.006**	**1.001**
n = 100	RB	-0.149	-0.018	-0.034	-0.064	-0.090	-0.108	-0.129	-0.137	-0.143	-0.147	-0.149
β = 1.5	RME	0.173	0.097	0.100	0.113	0.128	0.141	0.157	0.164	0.168	0.172	0.173
w = 3.0	EFF		**1.797**	**1.741**	**1.538**	**1.351**	**1.227**	**1.104**	**1.058**	**1.032**	**1.008**	**1.001**
n = 200	RB	-0.125	0.030	0.008	-0.029	-0.059	-0.080	-0.103	-0.113	-0.118	-0.123	-0.125
β = 1.5	RME	0.142	0.077	0.071	0.076	0.091	0.106	0.124	0.131	0.136	0.140	0.142
w = 3.0	EFF		**1.845**	**2.002**	**1.872**	**1.557**	**1.344**	**1.148**	**1.080**	**1.043**	**1.011**	**1.001**
n = 50	RB	-0.097	0.035	0.024	-0.006	-0.033	-0.053	-0.075	-0.084	-0.090	-0.095	-0.097
β = 2.0	RME	0.158	0.138	0.134	0.131	0.134	0.139	0.148	0.152	0.155	0.157	0.158
w = 2.5	EFF		**1.150**	**1.181**	**1.208**	**1.182**	**1.138**	**1.072**	**1.041**	**1.023**	**1.005**	**1.001**
n = 100	RB	-0.072	0.086	0.067	0.030	-0.002	-0.024	-0.049	-0.059	-0.065	-0.070	-0.072
β = 2.0	RME	0.121	0.134	0.122	0.106	0.101	0.103	0.110	0.115	0.117	0.120	0.121
w = 2.5	EFF		**0.902**	**0.991**	**1.147**	**1.204**	**1.180**	**1.100**	**1.058**	**1.033**	**1.008**	**1.001**
n = 200	RB	-0.061	0.126	0.098	0.052	0.016	-0.009	-0.036	-0.047	-0.053	-0.059	-0.061
β = 2.0	RME	0.094	0.147	0.123	0.091	0.076	0.074	0.081	0.086	0.090	0.093	0.094
w = 2.5	EFF		**0.639**	**0.764**	**1.032**	**1.239**	**1.270**	**1.161**	**1.092**	**1.052**	**1.013**	**1.001**
n = 50	RB	-0.053	0.094	0.082	0.048	0.018	-0.004	-0.029	-0.039	-0.045	-0.051	-0.053
β = 2.5	RME	0.138	0.165	0.157	0.141	0.133	0.131	0.133	0.135	0.136	0.138	0.138
w = 2.0	EFF		**0.839**	**0.881**	**0.979**	**1.038**	**1.054**	**1.041**	**1.026**	**1.015**	**1.003**	**1.000**
n = 100	RB	-0.041	0.136	0.113	0.071	0.036	0.012	-0.015	-0.026	-0.033	-0.039	-0.040
β = 2.5	RME	0.105	0.170	0.152	0.123	0.106	0.100	0.100	0.101	0.103	0.105	0.105
w = 2.0	EFF		**0.619**	**0.693**	**0.854**	**0.989**	**1.051**	**1.056**	**1.038**	**1.022**	**1.006**	**1.001**
n = 200	RB	-0.032	0.175	0.142	0.091	0.051	0.024	-0.006	-0.017	-0.024	-0.030	-0.032
β = 2.5	RME	0.084	0.194	0.164	0.122	0.095	0.083	0.079	0.080	0.081	0.083	0.084
w = 2.0	EFF		**0.432**	**0.510**	**0.686**	**0.881**	**1.011**	**1.065**	**1.049**	**1.030**	**1.008**	**1.001**

Table (2.8): The relative bias (RB), relative mean error (RME) and efficiency (EFF) of the proposed estimator $\bar{f}_m^*(0)$ with m = 2, 4, 7, 12, 20, 50, 100, 200, 1000 and 10000 when the data are simulated from Hazard-Rate (HR) Model.

Hazard-Rate Model	m	1	2	4	7	12	20	50	100	200	1000	10000
n = 50	RB	-0.417	-0.377	-0.382	-0.390	-0.397	-0.403	-0.409	-0.412	-0.414	-0.417	-0.417
β = 1.5	RME	0.432	0.395	0.399	0.407	0.414	0.419	0.425	0.427	0.429	0.431	0.432
w = 20.0	EFF		**1.094**	**1.082**	**1.061**	**1.044**	**1.032**	**1.017**	**1.011**	**1.006**	**1.002**	**1.000**
n = 100	RB	-0.374	-0.325	-0.332	-0.342	-0.350	-0.357	-0.364	-0.368	-0.370	-0.373	-0.374
β = 1.5	RME	0.382	0.336	0.342	0.352	0.360	0.366	0.374	0.377	0.379	0.381	0.382
w = 20.0	EFF		**1.137**	**1.116**	**1.086**	**1.062**	**1.044**	**1.023**	**1.014**	**1.008**	**1.002**	**1.000**
n = 200	RB	-0.324	-0.266	-0.274	-0.286	-0.296	-0.304	-0.313	-0.318	-0.320	-0.323	-0.324
β = 1.5	RME	0.329	0.272	0.280	0.292	0.302	0.310	0.319	0.323	0.325	0.328	0.329
w = 20.0	EFF		**1.208**	**1.174**	**1.126**	**1.089**	**1.063**	**1.033**	**1.020**	**1.012**	**1.003**	**1.000**
n = 50	RB	-0.269	-0.213	-0.215	-0.226	-0.236	-0.245	-0.256	-0.261	-0.264	-0.268	-0.269
β = 2.0	RME	0.296	0.253	0.255	0.263	0.270	0.277	0.285	0.289	0.292	0.295	0.296
w = 12.0	EFF		**1.171**	**1.161**	**1.127**	**1.095**	**1.069**	**1.037**	**1.022**	**1.013**	**1.003**	**1.000**
n = 100	RB	-0.226	-0.159	-0.162	-0.175	-0.188	-0.198	-0.211	-0.217	-0.221	-0.224	-0.226
β = 2.0	RME	0.242	0.186	0.189	0.200	0.210	0.218	0.229	0.235	0.238	0.241	0.242
w = 12.0	EFF		**1.302**	**1.280**	**1.214**	**1.155**	**1.110**	**1.056**	**1.033**	**1.019**	**1.005**	**1.001**
n = 200	RB	-0.179	-0.098	-0.103	-0.120	-0.135	-0.147	-0.163	-0.169	-0.174	-0.178	-0.179
β = 2.0	RME	0.191	0.123	0.128	0.140	0.153	0.163	0.177	0.183	0.186	0.190	0.191
w = 12.0	EFF		**1.553**	**1.500**	**1.366**	**1.252**	**1.172**	**1.084**	**1.048**	**1.027**	**1.007**	**1.001**
n = 50	RB	-0.126	-0.041	-0.042	-0.059	-0.076	-0.090	-0.107	-0.115	-0.120	-0.124	-0.125
β = 2.5	RME	0.179	0.150	0.150	0.152	0.156	0.161	0.169	0.173	0.175	0.178	0.179
w = 8.0	EFF		**1.189**	**1.192**	**1.180**	**1.149**	**1.113**	**1.061**	**1.036**	**1.020**	**1.005**	**1.001**
n = 100	RB	-0.097	0.003	0.000	-0.021	-0.042	-0.058	-0.077	-0.086	-0.091	-0.096	-0.097
β = 2.5	RME	0.130	0.099	0.097	0.097	0.101	0.107	0.118	0.123	0.126	0.129	0.130
w = 8.0	EFF		**1.320**	**1.339**	**1.349**	**1.291**	**1.215**	**1.109**	**1.062**	**1.035**	**1.009**	**1.001**
n = 200	RB	-0.067	0.055	0.048	0.021	-0.005	-0.023	-0.046	-0.055	-0.060	-0.066	-0.067
β = 2.5	RME	0.094	0.092	0.087	0.073	0.069	0.072	0.081	0.086	0.089	0.093	0.094
w = 8.0	EFF		**1.021**	**1.079**	**1.278**	**1.363**	**1.312**	**1.166**	**1.094**	**1.052**	**1.013**	**1.002**
n = 50	RB	-0.059	0.051	0.050	0.027	0.003	-0.015	-0.037	-0.046	-0.052	-0.057	-0.059
β = 3.0	RME	0.135	0.147	0.144	0.133	0.128	0.127	0.129	0.131	0.133	0.134	0.135
w = 6.0	EFF		**0.919**	**0.936**	**1.011**	**1.056**	**1.064**	**1.045**	**1.028**	**1.016**	**1.004**	**1.000**
n = 100	RB	-0.040	0.093	0.086	0.056	0.028	0.007	-0.017	-0.027	-0.033	-0.038	-0.040
β = 3.0	RME	0.100	0.138	0.132	0.112	0.100	0.095	0.095	0.096	0.098	0.099	0.100
w = 6.0	EFF		**0.724**	**0.759**	**0.893**	**1.006**	**1.057**	**1.058**	**1.038**	**1.023**	**1.006**	**1.001**
n = 200	RB	-0.018	0.142	0.128	0.090	0.058	0.034	0.007	-0.003	-0.010	-0.016	-0.017
β = 3.0	RME	0.069	0.160	0.147	0.110	0.090	0.076	0.068	0.067	0.067	0.068	0.069
w = 6.0	EFF		**0.429**	**0.468**	**0.600**	**0.766**	**0.903**	**1.015**	**1.025**	**1.019**	**1.006**	**1.001**

Table (2.9): The relative bias (RB), relative mean error (RME) and efficiency (EFF) of the proposed estimator $\tilde{f}^*_m(0)$ with m = 2, 4, 7, 12, 20, 50, 100, 200, 1000 and 10000 when the data are simulated from Beta (BE) Model.

Beta Model	m	1	2	4	7	12	20	50	100	200	1000	10000
n = 50	RB	-0.191	-0.073	-0.086	-0.113	-0.136	-0.153	-0.172	-0.180	-0.185	-0.189	-0.191
β = 1.5	RME	0.221	0.138	0.144	0.161	0.178	0.191	0.206	0.212	0.216	0.220	0.221
w = 1.0	EFF		**1.602**	**1.531**	**1.368**	**1.241**	**1.158**	**1.073**	**1.040**	**1.022**	**1.005**	**1.001**
n = 100	RB	-0.165	-0.024	-0.043	-0.076	-0.104	-0.123	-0.145	-0.154	-0.159	-0.164	-0.165
β = 1.5	RME	0.186	0.092	0.099	0.117	0.136	0.151	0.169	0.176	0.180	0.184	0.186
w = 1.0	EFF		**2.010**	**1.884**	**1.591**	**1.366**	**1.231**	**1.103**	**1.056**	**1.031**	**1.007**	**1.001**
n = 200	RB	-0.149	0.018	-0.009	-0.050	-0.081	-0.103	-0.127	-0.136	-0.142	-0.147	-0.148
β = 1.5	RME	0.164	0.075	0.073	0.087	0.108	0.125	0.145	0.153	0.158	0.162	0.164
w = 1.0	EFF		**2.187**	**2.249**	**1.877**	**1.516**	**1.311**	**1.132**	**1.071**	**1.039**	**1.009**	**1.001**
n = 50	RB	-0.212	-0.103	-0.114	-0.139	-0.160	-0.176	-0.194	-0.202	-0.206	-0.210	-0.212
β = 2.0	RME	0.238	0.153	0.160	0.179	0.195	0.208	0.223	0.229	0.233	0.237	0.238
w = 1.0	EFF		**1.555**	**1.484**	**1.331**	**1.216**	**1.142**	**1.067**	**1.037**	**1.021**	**1.005**	**1.001**
n = 100	RB	-0.191	-0.060	-0.077	-0.108	-0.133	-0.151	-0.171	-0.180	-0.185	-0.189	-0.191
β = 2.0	RME	0.209	0.108	0.118	0.140	0.160	0.175	0.192	0.199	0.204	0.208	0.209
w = 1.0	EFF		**1.936**	**1.770**	**1.495**	**1.308**	**1.196**	**1.089**	**1.049**	**1.027**	**1.007**	**1.001**
n = 200	RB	-0.169	-0.013	-0.038	-0.076	-0.105	-0.126	-0.148	-0.157	-0.162	-0.167	-0.169
β = 2.0	RME	0.182	0.072	0.080	0.103	0.126	0.143	0.163	0.171	0.176	0.180	0.181
w = 1.0	EFF		**2.539**	**2.283**	**1.771**	**1.445**	**1.270**	**1.116**	**1.063**	**1.034**	**1.008**	**1.001**
n = 50	RB	-0.236	-0.133	-0.143	-0.166	-0.187	-0.202	-0.219	-0.226	-0.230	-0.235	-0.236
β = 2.5	RME	0.259	0.174	0.181	0.200	0.216	0.229	0.244	0.250	0.254	0.257	0.258
w = 1.0	EFF		**1.488**	**1.427**	**1.295**	**1.195**	**1.129**	**1.061**	**1.034**	**1.019**	**1.004**	**1.000**
n = 100	RB	-0.216	-0.092	-0.108	-0.137	-0.161	-0.178	-0.198	-0.205	-0.210	-0.215	-0.216
β = 2.5	RME	0.232	0.129	0.140	0.164	0.184	0.199	0.216	0.223	0.227	0.231	0.232
w = 1.0	EFF		**1.804**	**1.653**	**1.419**	**1.263**	**1.169**	**1.077**	**1.042**	**1.023**	**1.006**	**1.001**
n = 200	RB	-0.184	-0.036	-0.059	-0.095	-0.123	-0.142	-0.163	-0.172	-0.177	-0.182	-0.183
β = 2.5	RME	0.194	0.076	0.089	0.115	0.139	0.156	0.176	0.184	0.188	0.193	0.194
w = 1.0	EFF		**2.560**	**2.181**	**1.684**	**1.397**	**1.243**	**1.106**	**1.057**	**1.031**	**1.008**	**1.001**
n = 50	RB	-0.256	-0.157	-0.167	-0.190	-0.209	-0.223	-0.239	-0.246	-0.250	-0.254	-0.256
β = 3.0	RME	0.276	0.191	0.199	0.218	0.235	0.247	0.261	0.267	0.271	0.275	0.276
w = 1.0	EFF		**1.444**	**1.386**	**1.266**	**1.176**	**1.117**	**1.055**	**1.031**	**1.017**	**1.004**	**1.000**
n = 100	RB	-0.222	-0.103	-0.118	-0.146	-0.169	-0.185	-0.204	-0.212	-0.216	-0.220	-0.222
β = 3.0	RME	0.235	0.132	0.144	0.167	0.187	0.202	0.219	0.226	0.230	0.234	0.235
w = 1.0	EFF		**1.781**	**1.636**	**1.408**	**1.256**	**1.164**	**1.075**	**1.041**	**1.023**	**1.006**	**1.001**
n = 200	RB	-0.198	-0.057	-0.079	-0.113	-0.140	-0.158	-0.179	-0.187	-0.192	-0.197	-0.198
β = 3.0	RME	0.208	0.088	0.103	0.130	0.154	0.171	0.190	0.198	0.202	0.207	0.208
w = 1.0	EFF		**2.367**	**2.019**	**1.597**	**1.352**	**1.218**	**1.096**	**1.053**	**1.029**	**1.007**	**1.001**

Table (2.10): The simulated value of the relative mean error (RME) and the relative bias (RB) for estimators $\hat{f}_{-1}(0)$ and $\hat{f}_0(0)$ by using the bandwidth $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}}$.

	N	RME (RB) $\hat{f}_1(0)$	RME (RB) $\hat{f}_{-1}(0)$	RME (RB) $\hat{f}_0(0)$
EP Model				
$\beta = 1$, $w = 5$	50	0.36 (-0.35)	1.00 (-1.00)	0.85 (-0.85)
	100	0.33 (-0.32)	1.00 (-1.00)	0.91 (-0.91)
	200	0.29 (-0.29)	1.00 (-1.00)	0.96 (-0.96)
$\beta = 1.5$, $w = 3$	50	0.21 (-0.18)	1.00 (-1.00)	0.79 (-0.79)
	100	0.17 (-0.15)	1.00 (-1.00)	0.88 (-0.88)
	200	0.14 (-0.13)	1.00 (-1.00)	0.94 (-0.94)
$\beta = 2$, $w = 2.5$	50	0.15 (-0.09)	1.00 (-1.00)	0.75 (-0.75)
	100	0.12 (-0.08)	1.00 (-1.00)	0.86 (-0.86)
	200	0.09 (-0.06)	1.00 (-1.00)	0.93 (-0.93)
$\beta = 2.5$, $w = 2$	50	0.14 (-0.05)	1.00 (-1.00)	0.73 (-0.73)
	100	0.11 (-0.04)	1.00 (-1.00)	0.84 (-0.84)
	200	0.08 (-0.03)	1.00 (-1.00)	0.93 (-0.93)
Hazard Rate Model				
$\beta = 1.5$, $w = 20$	50	0.43 (-0.42)	1.00 (-1.00)	0.89 (-0.89)
	100	0.39 (-0.38)	1.00 (-1.00)	0.94 (-0.94)
	200	0.33 (-0.33)	1.00 (-1.00)	0.97 (-0.97)
$\beta = 2$, $w = 12$	50	0.29 (-0.26)	1.00 (-1.00)	0.85 (-0.85)
	100	0.24 (-0.22)	1.00 (-1.00)	0.92 (-0.92)
	200	0.19 (-0.18)	1.00 (-1.00)	0.96 (-0.96)
$\beta = 2.5$, $w = 8$	50	0.18 (-0.13)	1.00 (-1.00)	0.81 (-0.81)
	100	0.13 (-0.10)	1.00 (-1.00)	0.89 (-0.89)
	200	0.09 (-0.06)	1.00 (-1.00)	0.95 (-0.95)
$\beta = 3$, $w = 6$	50	0.14 (-0.06)	1.00 (-1.00)	0.77 (-0.77)
	100	0.10 (-0.04)	1.00 (-1.00)	0.87 (-0.87)
	200	0.07 (-0.02)	1.00 (-1.00)	0.94 (-0.94)
Beta Model				
$\beta = 1.5$, $w = 1$	50	0.22 (-0.19)	1.00 (-1.00)	0.78 (-0.78)
	100	0.19 (-0.17)	1.00 (-1.00)	0.87 (-0.87)
	200	0.17 (-0.15)	1.00 (-1.00)	0.94 (-0.94)
$\beta = 2$, $w = 1$	50	0.24 (-0.21)	1.00 (-1.00)	0.79 (-0.79)
	100	0.21 (-0.19)	1.00 (-1.00)	0.88 (-0.88)
	200	0.18 (-0.17)	1.00 (-1.00)	0.94 (-0.94)
$\beta = 2.5$, $w = 1$	50	0.26 (-0.23)	1.00 (-1.00)	0.80 (-0.80)
	100	0.23 (-0.21)	1.00 (-1.00)	0.89 (-0.88)
	200	0.20 (-0.19)	1.00 (-1.00)	0.95 (-0.95)
$\beta = 3$, $w = 1$	50	0.27 (-0.25)	1.00 (-1.00)	0.81 (-0.81)
	100	0.24 (-0.22)	1.00 (-1.00)	0.89 (-0.89)
	200	0.21 (-0.20)	1.00 (-1.00)	0.95 (-0.95)

CHAPTER THREE

AN APPLICATION

3.1 INTRODUCTION

This chapter contains an application for the proposed estimators using two real data sets, which are usually applied and mentioned in line transect sampling literature. The first set is the stakes data and the other is Hemingway data set.

3.2 STAKES DATA SET

The stakes data which were collected as a part of a larger study on line transect sampling are given in Burnham et. al. (1980). These data are used to estimate the density of wooden stakes in a given area. The perpendicular distances (in meters) are presented in Table (3.1). There were 150 stakes randomly distributed in an area of $L = 1000$ meters long within a distance of 20 meters from the line in such a way that the maximum sighting distance was known. An observer moves follow the transect line and out of 150 stakes, 68 stakes were detected using the line transect technique. The true pdf of $f(x)$ was unknown, but the true value of $f(0)$ was known and it equals 0.110294. Thus the actual density D of stakes was 37.5 stakes/ha. (where, 1 hectare ≡ 1 ha = $10^4 m^2$) and the true number of stakes was $N = 150$ distributed in 4 ha. Note that $D = \frac{n\,f(0)}{2L}$ and $N = AD$, where A is the sampled area.

The different estimators that developed in this thesis are used to estimate the abundance of stakes and the standard error for each estimator is computed by using the bootstrap technique with 200 iterations. The different estimators of stakes data are implemented by using three values for the smoothing parameter h; $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}}$, $h_1 = 1.2h$ and $h_2 = 0.8h$. Moreover, the inspection of stakes data show that the perpendicular distance $x = 31.31$ seems to be are outlier. Therefore, we consider the two cases; $n = 68$ (including the value $x = 31.31$) and $n = 67$ (excluding the value $x = 31.31$).

Table (3.3) gives the point estimators of $f(0)$ in the case $n = 68$. While Table (3.4) gives the same results when $n = 67$. As the estimator $\hat{f}(0)$ of $f(0)$ is computed, the density estimator is calculated by using the relationship, $\widehat{\mathrm{D}} = \frac{n\,f(0)}{2L}$ and $\widehat{N} = \mathrm{A}\widehat{\mathrm{D}}$.

As Tables (3.3) and (3.4) shown, the different proposed estimators that used in the simulation study are applied for stakes data. In addition, the proposed estimator (2.2) is also applied for some values of $p < 1$ (e.g. $p = -1, 0, 0.25, 0.5, 0.8$). We note here that these values are not implemented for estimator $\hat{f}_p(0)$ in the simulation study of Chapter 2, because –as we stated in Chapter 2– the corresponding estimator for $p < 1$ give a poor performance. By examing the results of Tables (3.3) and (3.4), it is clear that the estimated values of $\hat{f}_{-1}(0), \hat{f}_0(0), \hat{f}_{0.25}(0)$, $\hat{f}_{0.5}(0)$ and $\hat{f}_{0.8}(0)$ are away from the actual value of $f(0)$. In the other word, the performances of these estimators are very poor compared to other estimators and the changing of the bandwidth h (see Table 3.3) cannot improve their performances.

The four estimators, $\tilde{f}_{\mathrm{a}}^*(0), \tilde{f}_{\mathrm{b}}^*(0), \tilde{f}_{\mathrm{c}}^*(0)$ and $\tilde{f}_{\mathrm{d}}^*(0)$ give –in some sense– similar results for these data with some preference for estimator $\tilde{f}_{\mathrm{b}}^*(0)$. The same result is obtained in the simulation study.

A closed look to the results of Tables (3.3) and (3.4) the results show that the estimators $\tilde{f}_{\mathrm{a}}^*(0), \tilde{f}_{\mathrm{b}}^*(0)$, $\tilde{f}_4^*(0)$, $\tilde{f}_7^*(0)$ and $\tilde{f}_{20}^*(0)$ perform very well for these stakes data. It is worthwhile to mention here that these estimators are in somehow special cases of estimator $\tilde{f}_m^*(0)$ (equation 2.3).

Table (3.1): The perpendicular distances of wooden stakes data (Burnham et. al. 1980)

i	x_i	i	x_i	i	x_i	i	x_i
1	2.02	18	1.61	35	3.79	52	8.49
2	0.45	19	4.08	36	15.24	53	6.08
3	10.4	20	6.5	37	3.47	54	0.4
4	3.61	21	8.27	38	3.05	55	9.33
5	0.92	22	4.85	39	7.93	56	0.53
6	1.0	23	1.47	40	18.15	57	1.23
7	3.4	24	18.6	41	10.5	58	1.67
8	2.9	25	0.41	42	4.41	59	4.53
9	8.16	26	0.4	43	1.27	60	3.12
10	6.47	27	0.2	44	13.72	61	3.05
11	5.66	28	11.59	45	6.25	62	6.6
12	2.95	29	3.17	46	3.59	63	4.4
13	3.96	30	7.1	47	9.04	64	4.97
14	0.09	31	10.71	48	7.68	65	3.17
15	11.82	32	3.86	49	4.89	66	7.67
16	14.23	33	6.05	50	9.1	67	18.16
17	2.44	34	6.42	51	3.25	68	31.31

3.3 HEMINGWAY DATA SET

The second set of real data are the Hemingway data on African ungulates described in Burnham et. al. (1980). This study detected 73 animals distributed on 60 km of transect line length. The true density of these detected animals and the true pdf of the perpendicular distances are unknown. The bootstrap technique with 200 iterations is used to find the standard error of each estimator that used to estimate $f(0)$ of Hemingway data. Table (3.5) presents the point estimates of $f(0)$ and D using the proposed estimators and it contains the relative bias (RB) and relative mean error (RME) for point estimates of $f(0)$, where the density D unit in this table is animal/ha.

As we show in Table (3.5), the different proposed estimators that used in the simulation study are applied for Hemingway data.

The value of estimator $\hat{f}_p(0)$ also the dependent density D are increases as p increases in the interval of p [1,2]. Moreover, RME's of this estimator are decrease in this interval. But we not prefer this estimator to estimate the density of this data set because it produces an increasing SE as p increases.

The proposed estimators $\tilde{f}_c^*(0)$ and $\tilde{f}_d^*(0)$ perform more efficient results than the estimator $\hat{f}_1(0)$ with respect to RB, RME, and SE.

The estimator $\tilde{f}_m^*(0)$ produced more efficient results than the other proposed estimators specially when used some values of m based on the global measure RME. Finally, the estimator $\tilde{f}_{20}^*(0)$ have a good estimator and we say it to estimate $f(0)$ in this data set.

Table (3.2): The perpendicular distances of Hemingway data (Burnham et. al. 1980)

i	x_i	i	x_i	i	x_i	i	x_i
1	0	20	41	39	86	58	157
2	0	21	42.1	40	86	59	161
3	0	22	50.8	41	87	60	164
4	0	23	55.1	42	90	61	164
5	0	24	58.5	43	92.3	62	164
6	0	25	63.6	44	94	63	166
7	0	26	64.3	45	96.4	64	175
8	0	27	65	46	96.4	65	188
9	8.72	28	68.8	47	106	66	193
10	10.5	29	71.1	48	115	67	200
11	22.3	30	71.8	49	123	68	200
12	26	31	71.9	50	123	69	246
13	26	32	72.1	51	129	70	260
14	30	33	73.1	52	129	71	272
15	30.5	34	76.6	53	143	72	378
16	31.7	35	77.6	54	143	73	400
17	34.2	36	78.1	55	150		
18	35.1	37	84.5	56	151		
19	38	38	84.5	57	153		

Table (3.3): The point estimators of $f(0)$ for the stakes data set when the sample size is 68 and the true values of $f(0)$ is 0.110294

Estimator	$\hat{f}(0)$ when using $h = 1.06\,\hat{\sigma}\, n^{-\frac{1}{5}} = 3.754$	$\hat{f}(0)$ when using $h1 = 1.2\,h = 4.505$	$\hat{f}(0)$ when using $h2 = 0.8\,h = 3.003$
$\hat{f}_1(0)$	0.097578	0.093380	0.100953
$\hat{f}_{-1}(0)$	0.000000	0.000000	0.000000
$\hat{f}_0(0)$	0.019162	0.033311	0.006187
$\hat{f}_{0.25}(0)$	0.054495	0.061705	0.043004
$\hat{f}_{0.5}(0)$	0.073717	0.076081	0.067759
$\hat{f}_{0.8}(0)$	0.089492	0.087575	0.089487
$\hat{f}_{1.1}(0)$	0.101117	0.095908	0.106021
$\hat{f}_{1.3}(0)$	0.107411	0.100390	0.115094
$\hat{f}_{1.5}(0)$	0.112862	0.104262	0.123006
$\hat{f}_2(0)$	0.123848	0.112055	0.139102
$\tilde{f}_a^*(0)$	0.107924	0.100660	0.116133
$\tilde{f}_b^*(0)$	0.107553	0.100627	0.115066
$\tilde{f}_c^*(0)$	0.105861	0.100432	0.110430
$\tilde{f}_d^*(0)$	0.103228	0.098419	0.107320
$\tilde{f}_2^*(0)$	0.110272	0.102765	0.118717
$\tilde{f}_4^*(0)$	0.110204	0.103032	0.117584
$\tilde{f}_7^*(0)$	0.107455	0.101372	0.112754
$\tilde{f}_{20}^*(0)$	0.102035	0.097359	0.105976
$\tilde{f}_{50}^*(0)$	0.099661	0.095253	0.103251

Table (3.4): The point estimators of $f(0)$ for the stakes data set when the sample size is 67 and the true values of $f(0)$ is 0.110294

Estimator	$\hat{f}(0)$ when using $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}} = 3.365$	$\hat{f}(0)$ when using $h1 = 1.2\,h = 4.038$	$\hat{f}(0)$ when using $h2 = 0.8\,h = 2.692$
$\hat{f}_1(0)$	0.100804	0.097359	0.103323
$\hat{f}_{-1}(0)$	0.000002	0.000144	0.000000
$\hat{f}_0(0)$	0.021682	0.037529	0.007057
$\hat{f}_{0.25}(0)$	0.051987	0.061110	0.039046
$\hat{f}_{0.5}(0)$	0.073153	0.077081	0.065619
$\hat{f}_{0.8}(0)$	0.091316	0.090467	0.090126
$\hat{f}_{1.1}(0)$	0.104983	0.100381	0.109202
$\hat{f}_{1.3}(0)$	0.112447	0.105763	0.119787
$\hat{f}_{1.5}(0)$	0.118936	0.110433	0.129081
$\hat{f}_2(0)$	0.132079	0.119866	0.148167
$\hat{f}_a^*(0)$	0.113146	0.106137	0.121171
$\hat{f}_b^*(0)$	0.112582	0.106040	0.119787
$\hat{f}_c^*(0)$	0.109735	0.105203	0.113350
$\hat{f}_d^*(0)$	0.106832	0.102787	0.110040
$\hat{f}_2^*(0)$	0.115754	0.108544	0.124022
$\hat{f}_4^*(0)$	0.115403	0.108732	0.121826
$\hat{f}_7^*(0)$	0.111725	0.106519	0.115906
$\hat{f}_{20}^*(0)$	0.105558	0.101643	0.108627
$\hat{f}_{50}^*(0)$	0.103007	0.099369	0.105721

Table (3.5): The point estimators of $f(0)$, D, relative bias (RB), relative mean error (RME) and the standard error of each estimator of $f(0)$ for the Hemingway data set. The bandwidth $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}} = 58.209$, and $n = 73$. The true values of $f(0)$, D are unknown

Estimator	$\hat{f}(0)$	RB	RME	$\hat{D}$	$SE(\hat{f}(0))$
$\hat{f}_1(0)$	0.005725	-0.020878	0.024433	0.034827	0.000074
$\hat{f}_{1.1}(0)$	0.005969	-0.020797	0.024282	0.036313	0.000076
$\hat{f}_{1.3}(0)$	0.006407	-0.020599	0.023984	0.038978	0.000080
$\hat{f}_{1.5}(0)$	0.006790	-0.020378	0.023702	0.041306	0.000084
$\hat{f}_2(0)$	0.007571	-0.019811	0.023086	0.046055	0.000092
$\tilde{f}_a^*(0)$	0.006450	-0.020564	0.023933	0.039236	0.000081
$\tilde{f}_b^*(0)$	0.006422	-0.020249	0.023631	0.039069	0.000080
$\tilde{f}_c^*(0)$	0.006237	-0.018361	0.021990	0.037942	0.000077
$\tilde{f}_d^*(0)$	0.006081	-0.018516	0.022224	0.036993	0.000076
$\tilde{f}_2^*(0)$	0.006618	-0.019643	0.023042	0.040260	0.000081
$\tilde{f}_4^*(0)$	0.006571	-0.018329	0.021788	0.039972	0.000079
$\tilde{f}_7^*(0)$	0.006340	-0.018319	0.021864	0.038565	0.000077
$\tilde{f}_{20}^*(0)$	0.006007	-0.019056	0.022721	0.036542	0.000076
$\tilde{f}_{50}^*(0)$	0.005848	0.021042	0.024767	0.035574	0.000075

CHAPTER FOUR

CONCLUSIONS AND FUTURE WORK

4.1 OVERVIEW

In this chapter, we will summarize the conclusions of this thesis. In addition, we note some remarks and give research ideas for further studies.

4.2 CONCLUSIONS

On the basis of the previous two chapters we summarize the following conclusions:

(1) The variation of the smoothing parameter (bandwidth) h increasing or decreasing improve the performances of the different estimators in some cases but in the other cases their performances become worse (i.e. the bandwidth selections have not played a critical role in improving the kernel estimator $\hat{f}_p(0)$). Therefore, we generally recommend to use the rule $h = 1.06\,\hat{\sigma}\,n^{-\frac{1}{5}}$ without any modifications of this rule when we estimate $f(0)$.

(2) The value of estimator $\hat{f}_p(0)$ is very sensitive to the choice of p.

(3) The value of estimator $\hat{f}_p(0)$ increases as p increases in the interval $(1 \le p \le 2)$.

(4) The performances of estimator $\hat{f}_p(0)$ are satisfactory for $1 \le p \le 2$. In the other hand, the performance of $\hat{f}_p(0)$ becomes very poor as we take p away below 1 or away above 2.

(5) The RMEs of $\hat{f}_2(0)$ are –generally– not decreasing when n increases, which indicates that $\hat{f}_2(0)$ is not even a consistent estimator for $f(0)$.

(6) Also, we advise to exclude poor performance estimators from the future researches; in particularly $\hat{f}_{-1}(0)$, and $\hat{f}_0(0)$. Moreover, the estimator $\hat{f}_p(0)$ has given unsuitable performance in the cases of p lie out of the range [1,2].

(7) The estimator $\tilde{f}^*_{\mathrm{m}}(0)$ is surprising in most considered cases. Therefore, we can be implemented to use this estimator in line transect sampling to estimate $f(0)$ and consequently to estimate the abundance D according to the following rule:

a. If the collected perpendicular distances $x_1, x_2, \ldots x_n$ haven't a shoulder at the origin then $\tilde{f}^*_{\mathrm{m}}(0)$ with m around the value 4 can be used.

b. If the perpendicular distances seem to be have a shoulder condition at the origin then $\tilde{f}^*_{\mathrm{m}}(0)$ with a value of m around 15 can be used.

c. If there is no information about the model shape of the perpendicular distances then we recommend to use $\tilde{f}^*_{\mathrm{m}}(0)$ with a value of m around 10.

(8) The estimators $\tilde{f}^*_{\mathrm{a}}(0), \tilde{f}^*_{\mathrm{b}}(0), \tilde{f}^*_{\mathrm{c}}(0)$ and $\tilde{f}^*_{\mathrm{d}}(0)$ give very similar performances with overall a slight preference for $\tilde{f}^*_{\mathrm{b}}(0)$. In general, their performances are acceptable comparing to that of $\hat{f}_1(0)$.

(9) At the end, the use of the nonparametric proposed estimators (specially the estimator $\tilde{f}^*_{\mathrm{m}}(0)$) may be recommended in line transect studies since they turns out to be accurate and robust for several cases considered in the sense that it shows stable performance for very different models.

4.3 FUTURE WORK

There are various possibilities for future research, some suggestions would include:

(1) The surprise results of the proposed estimator $\tilde{f}_m^*(0)$ which are leading to interest and to derive the mathematical properties of their, since they have good statistical properties. So that, we still need more work to enhance this estimators in future.

(2) The performances of the proposed estimator $\tilde{f}_m^*(0)$ can be changed when using other correction factors which need to study and to compare the effect of these new correction factors.

(3) The estimator $\hat{f}_p(0)$ have various performances in some models. Therefore, we advise to use it in estimating $f(0)$ through other estimator like $\tilde{f}_m^*(0)$.

(4) However, the proposed estimators could be use in other life fields needing to estimate $f(0)$; particularly in the wildlife studies.

References

- Barabesi, L. (2000). Local likelihood density estimation in line transect sampling. Environmetrics, 11: 413-422.
- Barabesi, L., Greco, L. and Naddeo, S. (2002). Density estimation in line transect sampling with grouped data by local least squares. Environmetrics, 13: 167-176.
- Buckland, S.T. (1985). Perpendicular distance model for line transect sampling. Biometrics, 41: 177-195.
- Buckland, S.T. (1992). Fitting density function with polynomials. Applied Statistics, 41: 63-76.
- Buckland, S.T., Anderson, D. R., Burnham, K. P., Laake, J. L., Borchers, D.L. & Thomas, L. (2001). Introduction to Distance Sampling. Oxford University Press, London.
- Burnham, K. P. and Anderson, D. R. (1976). Mathematical models for nonparametric influences from line transect data. Biometrics, 32: 325-336.
- Burnham, K. P., Anderson, D. R., and Laake, J. L. (1980). Estimation of density from line transect sampling of biological populations. Wildlife Monographs, 72: 1-202
- Chen, S. X. (1996a). A kernel estimate for the density of a biological population by using line transects sampling. Applied Statistics, 45, No. 2: 135-150.
- Chen, S.X. (1996b). Studying school size effects in line transect sampling using the kernel method. Biometrics, 52: 1283-1294.
- Cline, D.B.H. (1988). Admissible kernel estimators of a multivariate density. The Annals of Statistics, 16: 1421-1427
- Eberhardt, L. L. (1968). A preliminary appraisal of line transects. J. Wildl. Manage. 32: 82-88.
- Eidous, O.M. (2005). On improving kernel estimators using line transect sampling. Communications in Statistics - Theory and Methods, 34: 931-941.
- Eidous, O.M. (2011). Variable location kernel method using line transect sampling. Environmetrics, 22: 431-440.
- Eidous, O.M., and Shakhatreh, M. (2011). Asymptotic unbiased kernel estimator for line transect sampling. To appear in Communications in Statistics - Theory and Methods.
- Fewster, R.M., Laake, J.L., and Buckland, S.T. (2005). Line transect sampling in small and large regions. Biometrics, 61: 856–859.
- Fix, E. and Hodges, J. L. Jr. (1951). Discriminatory analysis, nonparametric discrimination: Consistency properties. Report No. 21-49-004, USAF School of Aviation Medicine.

- Gates, C.E., Marshall, W.H., and Olson, D.P. (1968). Line transect method of estimating grouse population densities. Biometrics, 24: 135-145.
- Gerard, P.D. and Schucany, W.R. (1999). Local bandwidth selection for kernel estimation of population densities with line transect sampling. Biometrics, 55: 769-773.
- Gong, R. , Ma, Y. , and Zhou, Y. (2010). Confident Estimation for Density of a Biological Population Based on Line Transect Sampling. Acta Mathematicae Applicatae Sinica, 26, No. 1: 79-92.
- Hayes, R. J., and Buckland, S. T. (1983). Radial distance models for line-transect method. Biometrics, 39: 29-42.
- Hemingway, P. (1971). Field trials of the line transect method of sampling populations of larger herbivores. In The scientific management of animal and plant communities for conservation, Eds E. Duffey & A. S. Watt, 405-411. Oxford: Blackwell Scientific Publications.
- Hiby, L., and Krishna, M.B. (2001). Line transect sampling from a curving path. Biometrics, 57: 727-731.
- Mack, Y. P., (1998). Testing for the shoulder condition in transect sampling. Communications in Statistics - Theory and Methods, 27, No. 2: 423-432.
- Mack, Y. P. (2002). Bias corrected confidence intervals for wildlife abundance estimation. Communications in Statistics - Theory and Methods, 31: 1107-1122.
- Mack, Y. P., Quang, P.X. and Zhang, S., (1999). Kernel estimation in transect sampling without the shoulder condition. Communications in Statistics - Theory and Methods. 28: 2277-2296.
- Marques, T.A. (2004). Predicting and correcting bias caused by measurement error in line transect sampling using multiplicative error models. Biometrics, 60: 757-763.
- Marshall, A.R., Lovett, J.C., White, P.C. (2008). Selection of line-transect methods for estimating the density of group-living animals: lessons from the primates. American Journal of Primatology, 70: 452-462.
- Melville, G.J., Welsh, A.H. (2001). Line transect sampling in small regions. Biometrics, 57: 1130-1137.
- Okamura, H. (2003). A line transect method to estimate abundance of long-diving animals. Fisheries Science, 69: 1176-1181.
- Pollock, K. H. (1978). A family of density estimators for line transect sampling. Biometrics, 34: 475-478.
- Quang, P.X. (1990). Confidence intervals for densities in line transect sampling. Biometrics, 46: 459-472.
- Quang, P.X. (1993). Nonparametric estimators for variable circular plot surveys. Biometrics, 49: 837-852.

- Seber, G.A.F. (1982). The estimation of animal abundance and related parameters, 2nd ed. Charles Griffin & Company Ltd., London
- Silverman, B. W. (1986). Density estimation for statistics and data analysis. Chapman and Hall, London, New York.
- Wand, M. P. and Jones, M. C. (1995). Kernel smoothing. Chapman and Hall, London
- Zhang, S. (2001). Generalized likelihood ratio test for the shoulder condition in line transect sampling. Communications in Statistics - Theory and Methods, 30: 2343-2354.